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The Interpretation of Pumping Tests in Aquifers with Linear Boundaries

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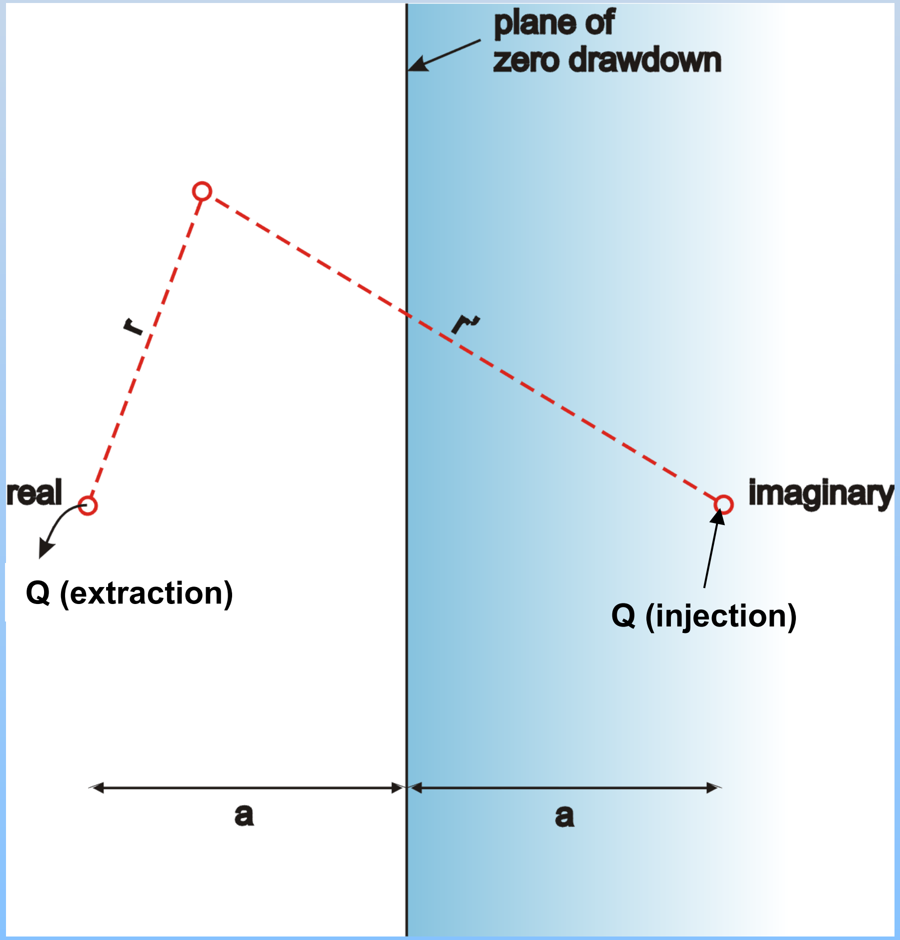
Simplified conceptual models

1. Linear constant-head boundary
2. Linear no-flow boundary
3. Two linear no-flow boundaries (strip)

   → Buried channel aquifers

Model 1:
Linear, constant-head boundary



Theis solution with an image well:

$$s(r,t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) - \frac{Q}{4\pi T} W\left(\frac{r'^2 S}{4Tt}\right)$$

Cooper-Jacob approximation:

$$s(r,t) \approx \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r^2 S}{4Tt}\right\}\right] - \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r'^2 S}{4Tt}\right\}\right]$$

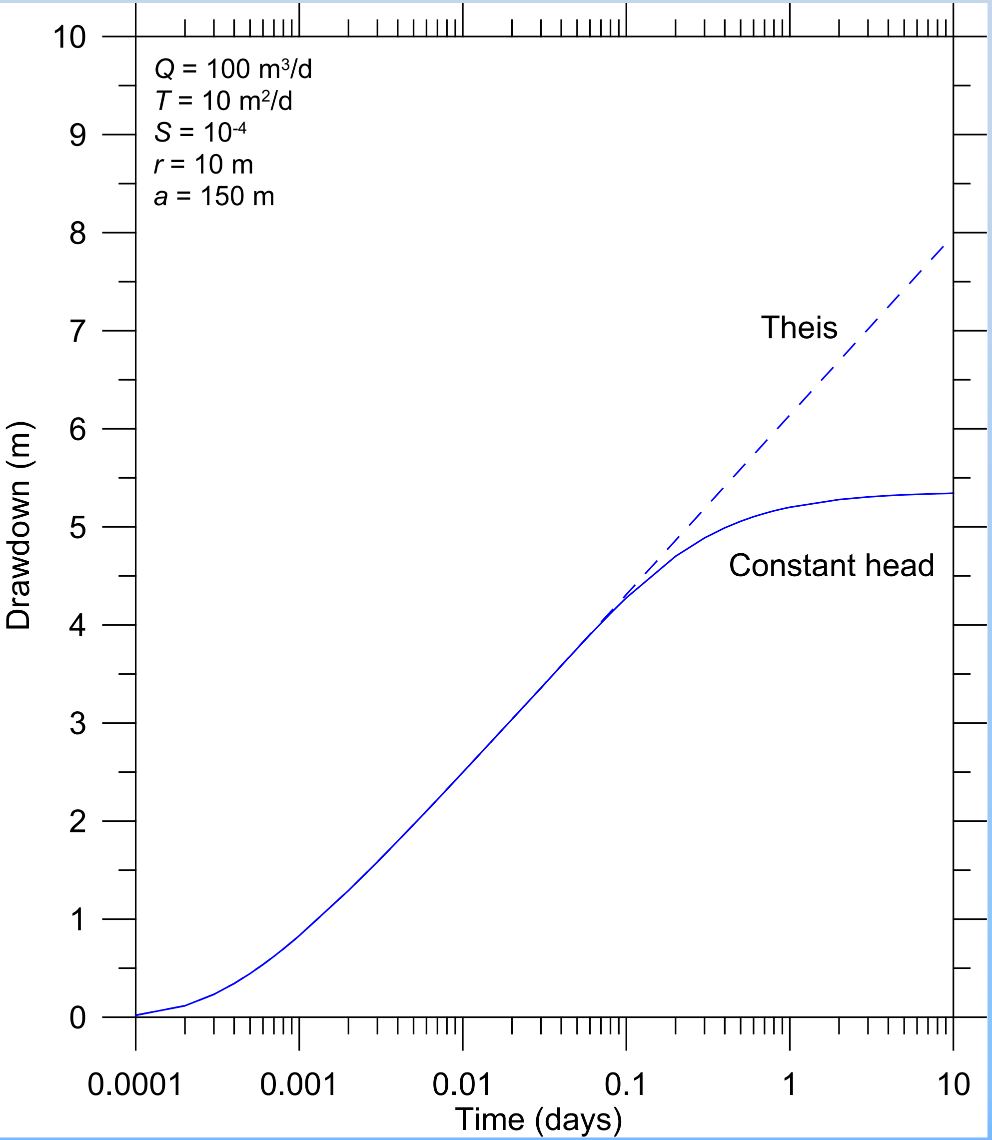
Cooper-Jacob approximation (2):

$$s(r,t) \approx \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r^2 S}{4Tt}\right\}\right] - \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r'^2 S}{4Tt}\right\}\right]$$

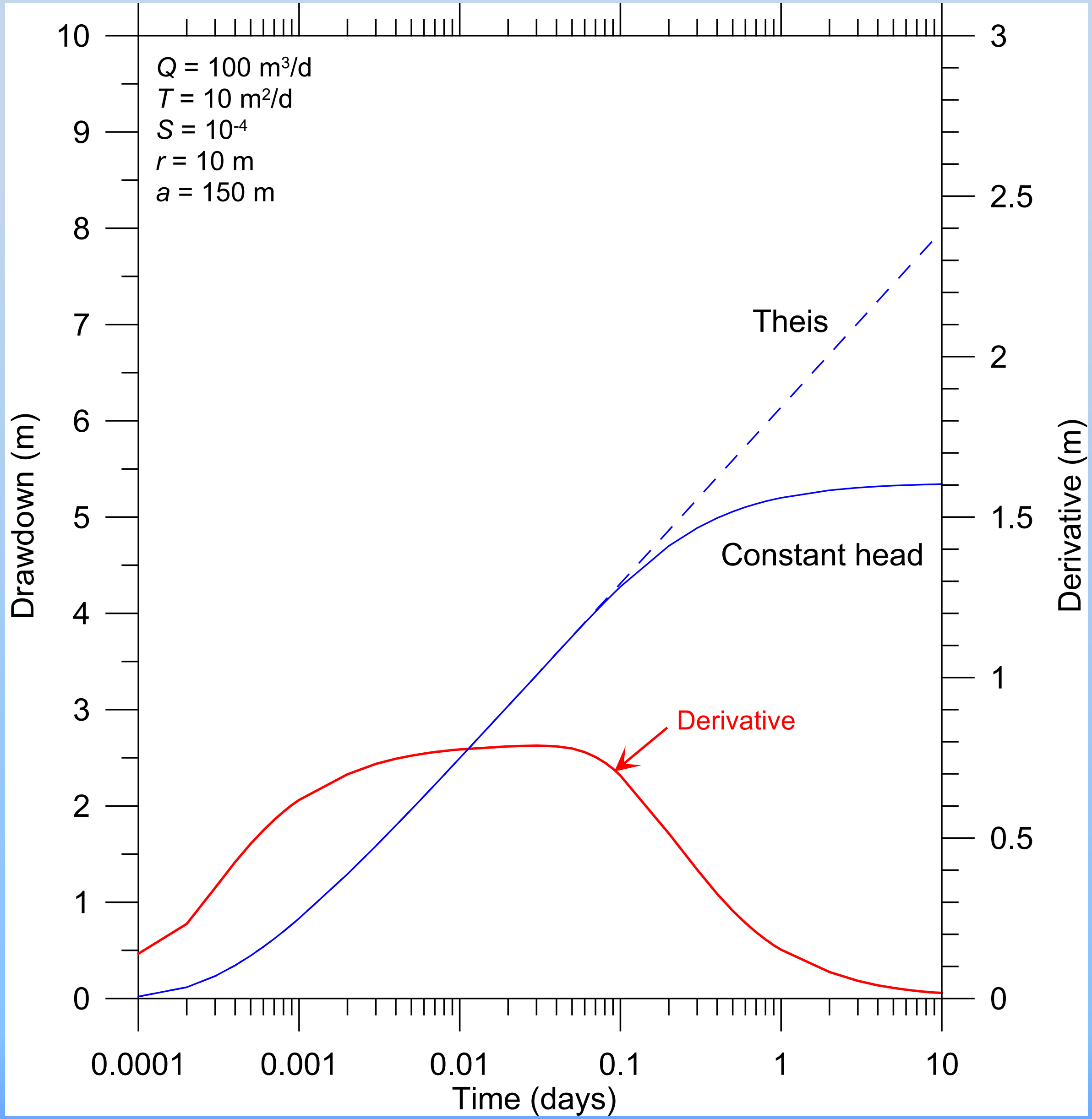
$$= \frac{Q}{4\pi T}\left(\left[-ln\{r^2\} - ln\left\{\frac{S}{4Tt}\right\}\right] + \left[ln\{r'^2\} + ln\left\{\frac{S}{4Tt}\right\}\right]\right)$$

$$= \frac{Q}{4\pi T} ln\left\{\frac{r'^2}{r^2}\right\}$$

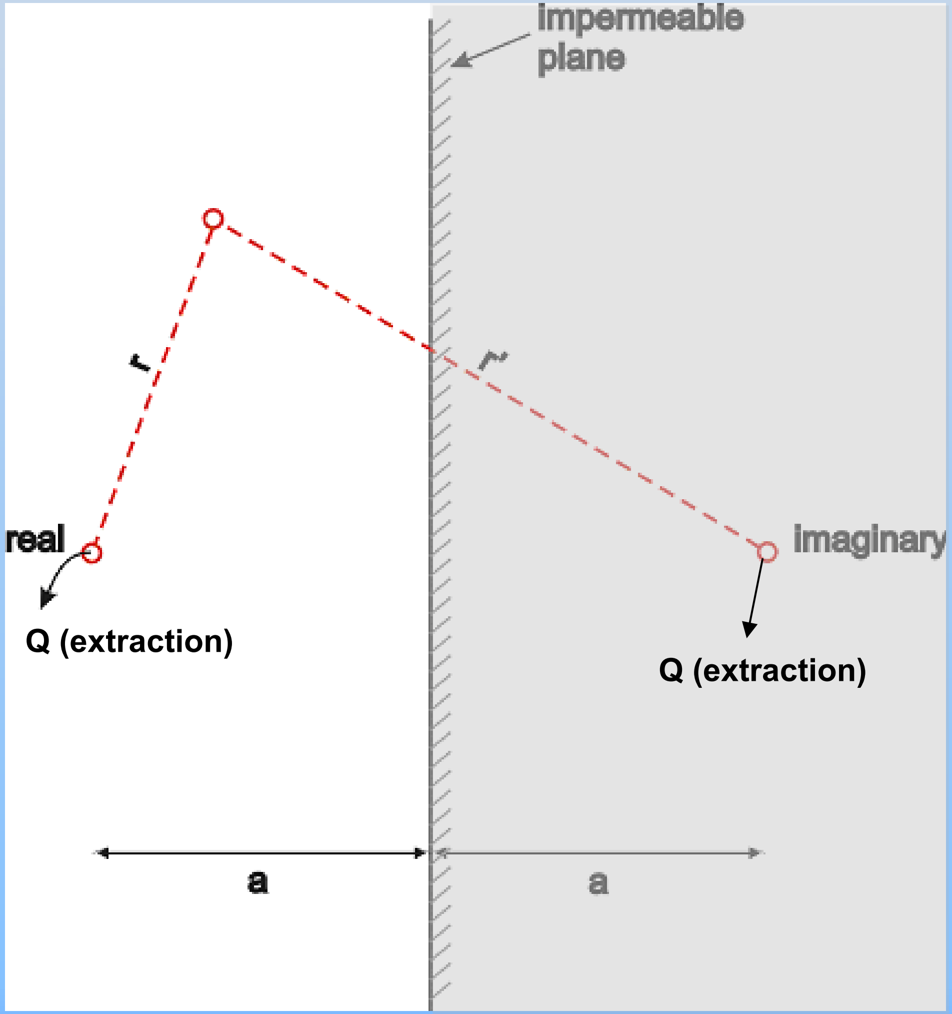
Semi-log plot



Semi-log plot with derivative



Model 2:
Linear, impermeable boundary



Theis solution with an image well:

$$s(r,t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) + \frac{Q}{4\pi T} W\left(\frac{r'^2 S}{4Tt}\right)$$

Cooper-Jacob approximation:

$$s \approx \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r^2 S}{4Tt}\right\}\right] + \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r'^2 S}{4Tt}\right\}\right]$$

Cooper-Jacob approximation (2)

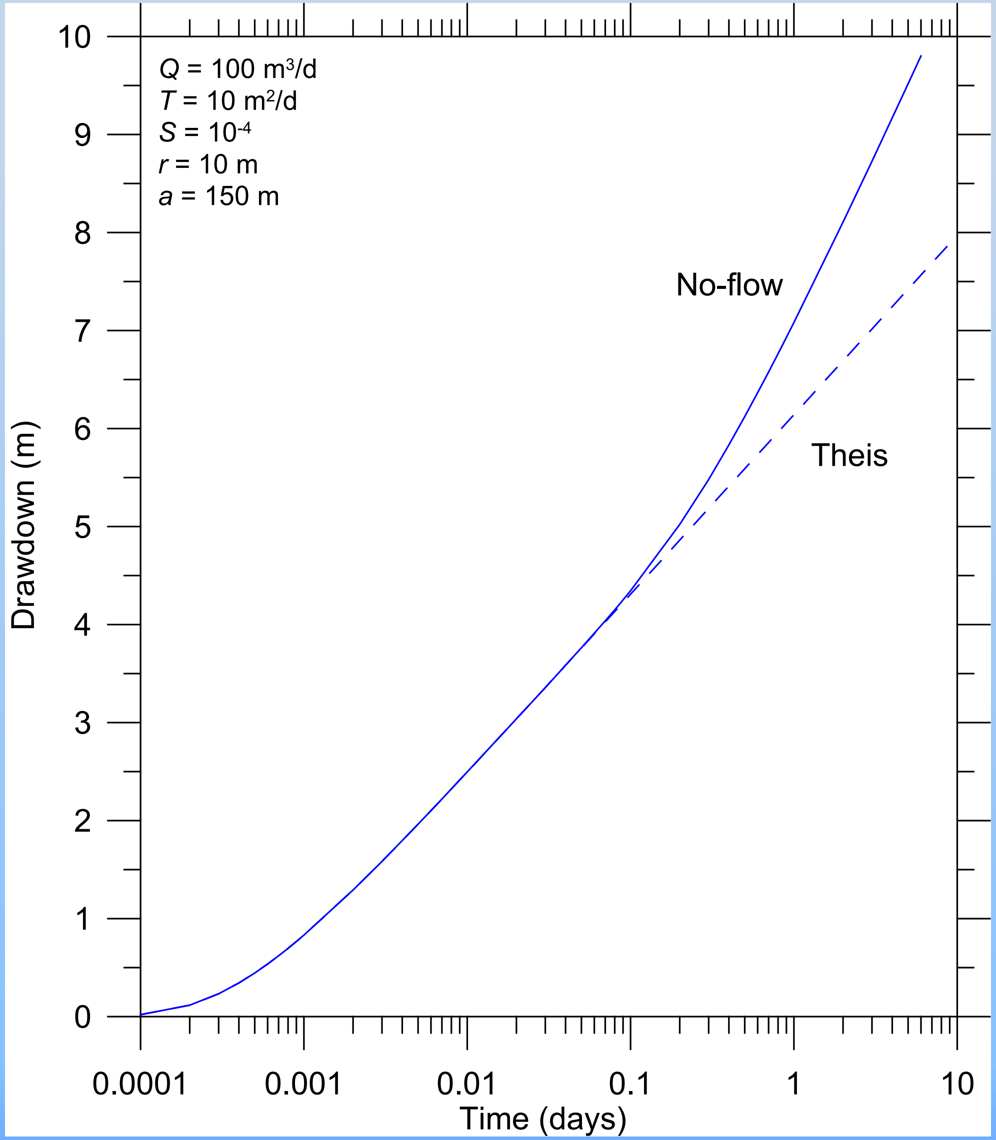
$$s \approx \frac{Q}{4\pi T}\left[\left(-0.5772 - ln\left\{\frac{r^2 S}{4Tt}\right\}\right) + \left(-0.5772 - ln\left\{\frac{r'^2 S}{4Tt}\right\}\right)\right]$$

$$= \frac{Q}{4\pi T}\left[2(-0.5772) - 2\ ln\left\{\frac{S}{4Tt}\right\} - 2\ ln\{r\} - 2\ ln\{r'\}\right]$$

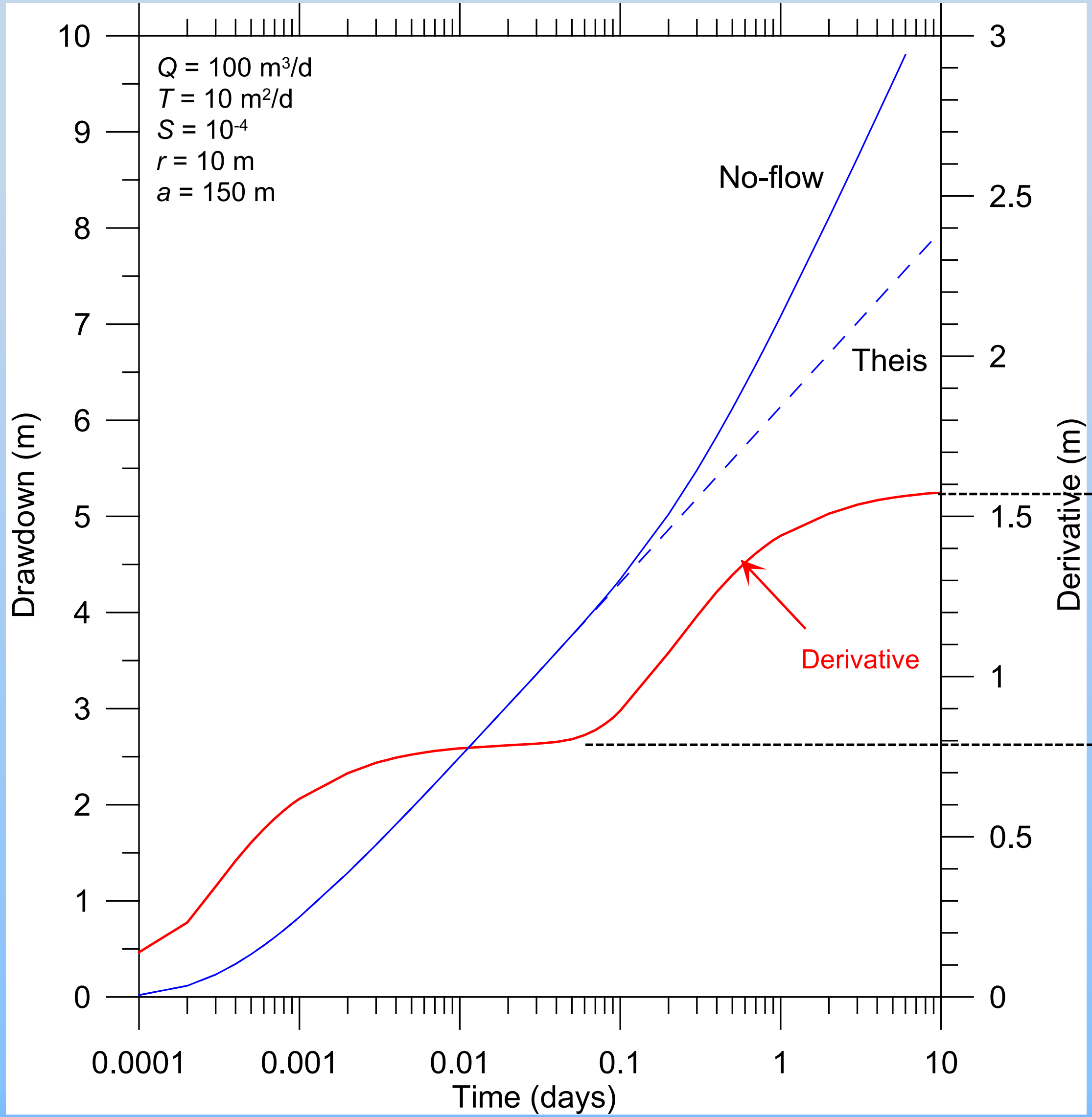
$$= \frac{Q}{4\pi T}\left[2\ ln\{0.561\} + 2\ ln\left\{\frac{4Tt}{S}\right\} + 2\ ln\left\{\frac{1}{rr'}\right\}\right]$$

$$= \frac{Q}{2\pi T} 2.303\left(\log_{10}\left\{2.2459\frac{T}{S}\frac{1}{rr'}\right\} + \log_{10}\{t\}\right)$$

Semi-log plot



Semilog plot with derivative

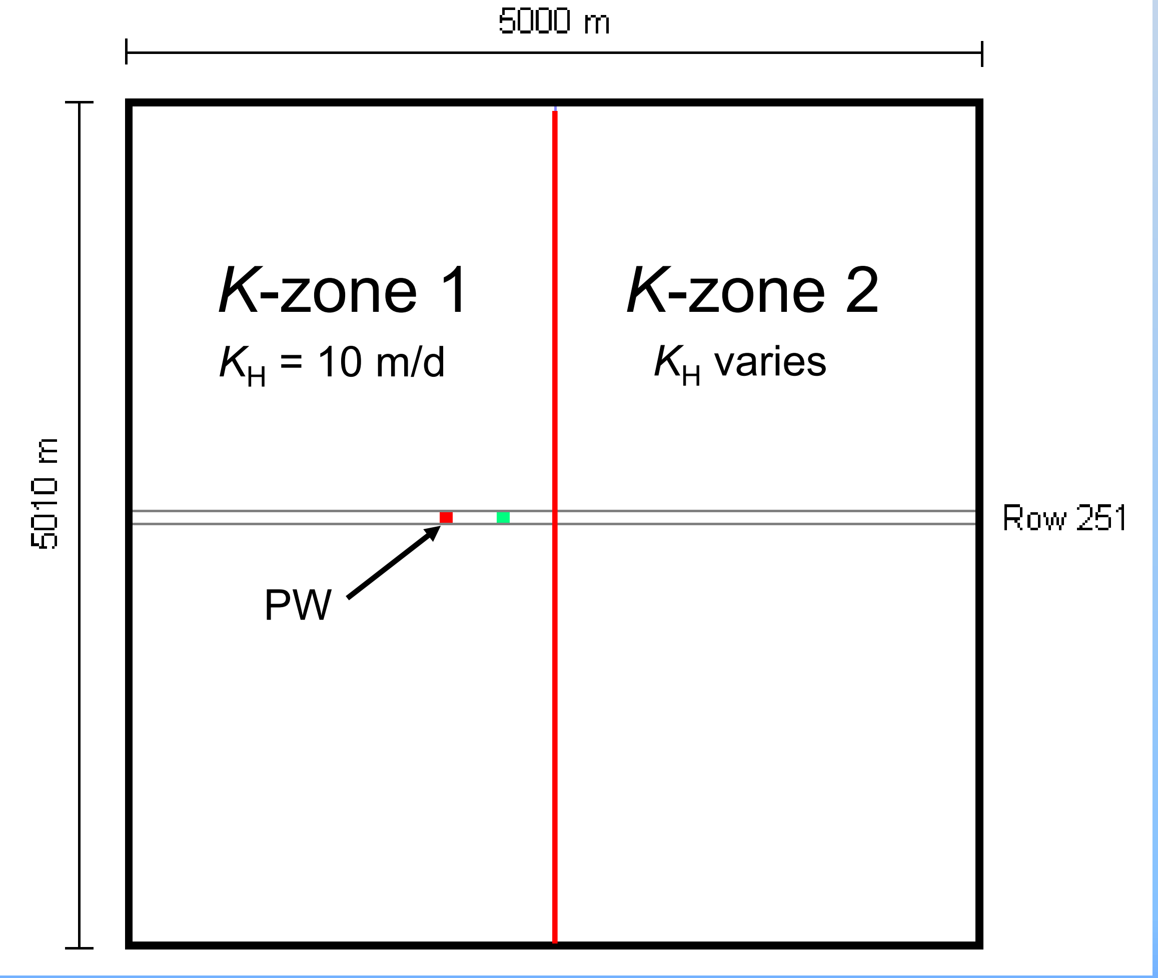


D_t = 1.56 m

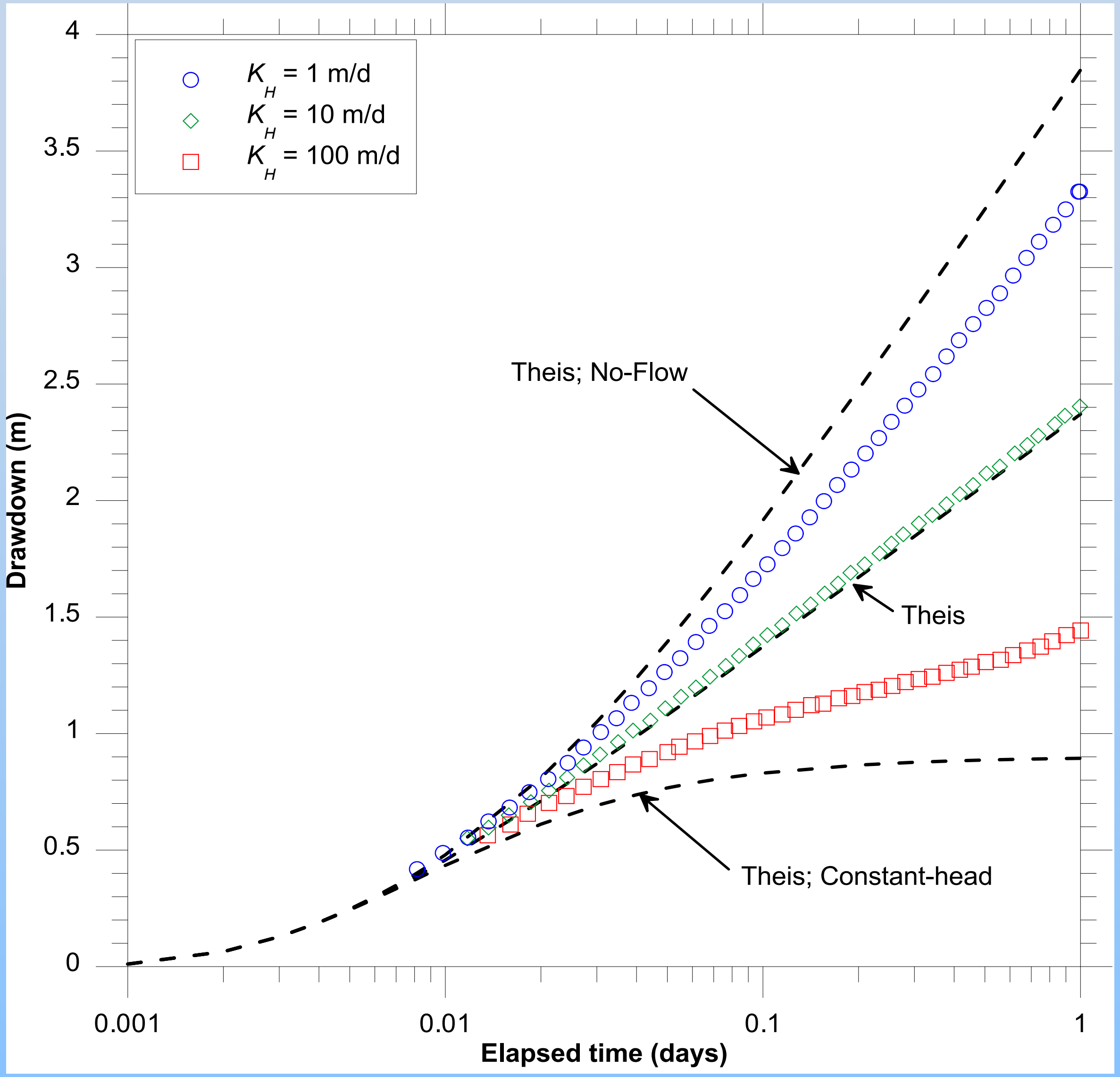
D_t = 0.78 m

Diagnosis of an “imperfect” linear boundary

Example analysis for a 2-zone aquifer



Zone 1: K_H = 10 m/d



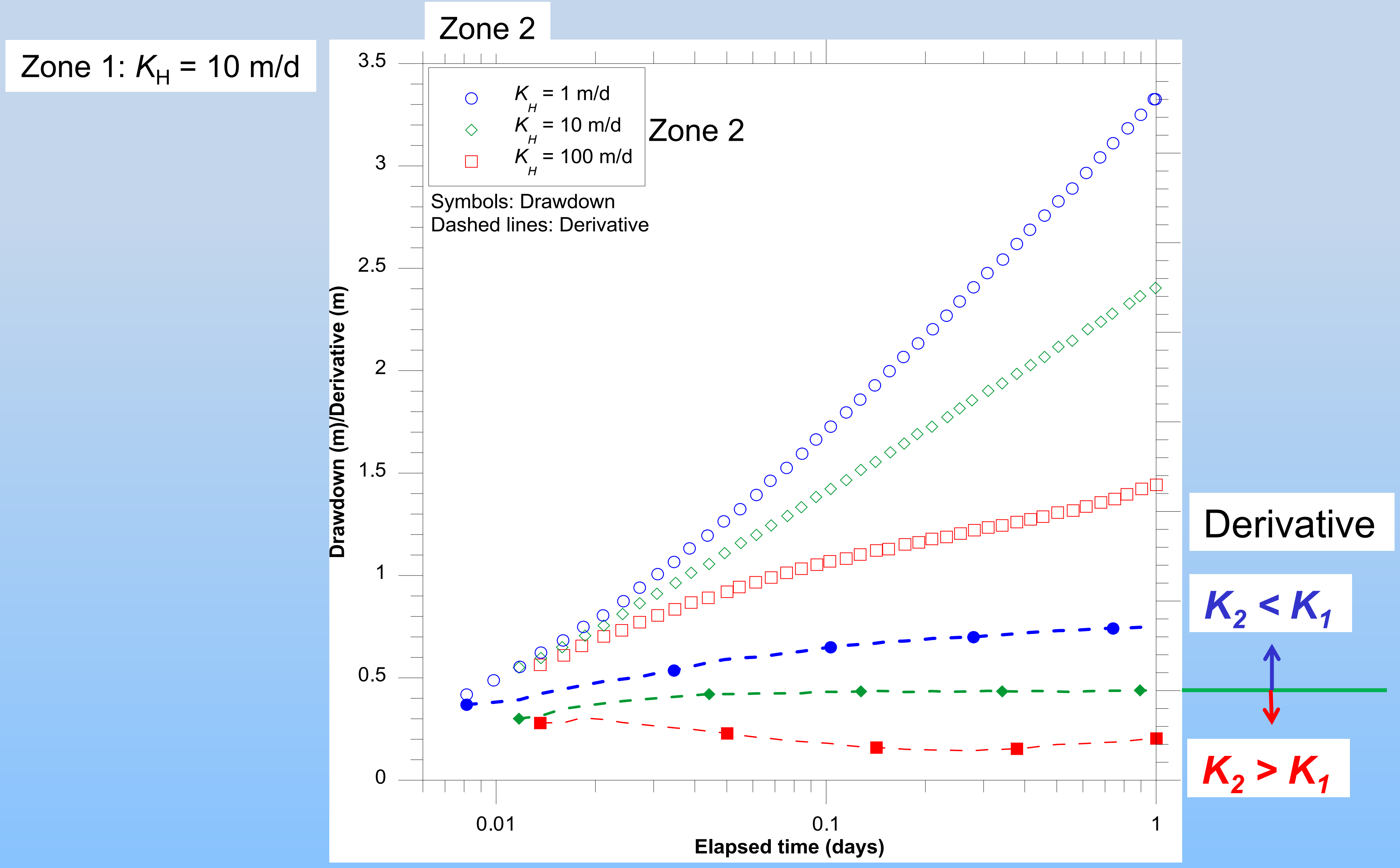
Zone 2

$K_2 = 0$

$K_2 < K_1$

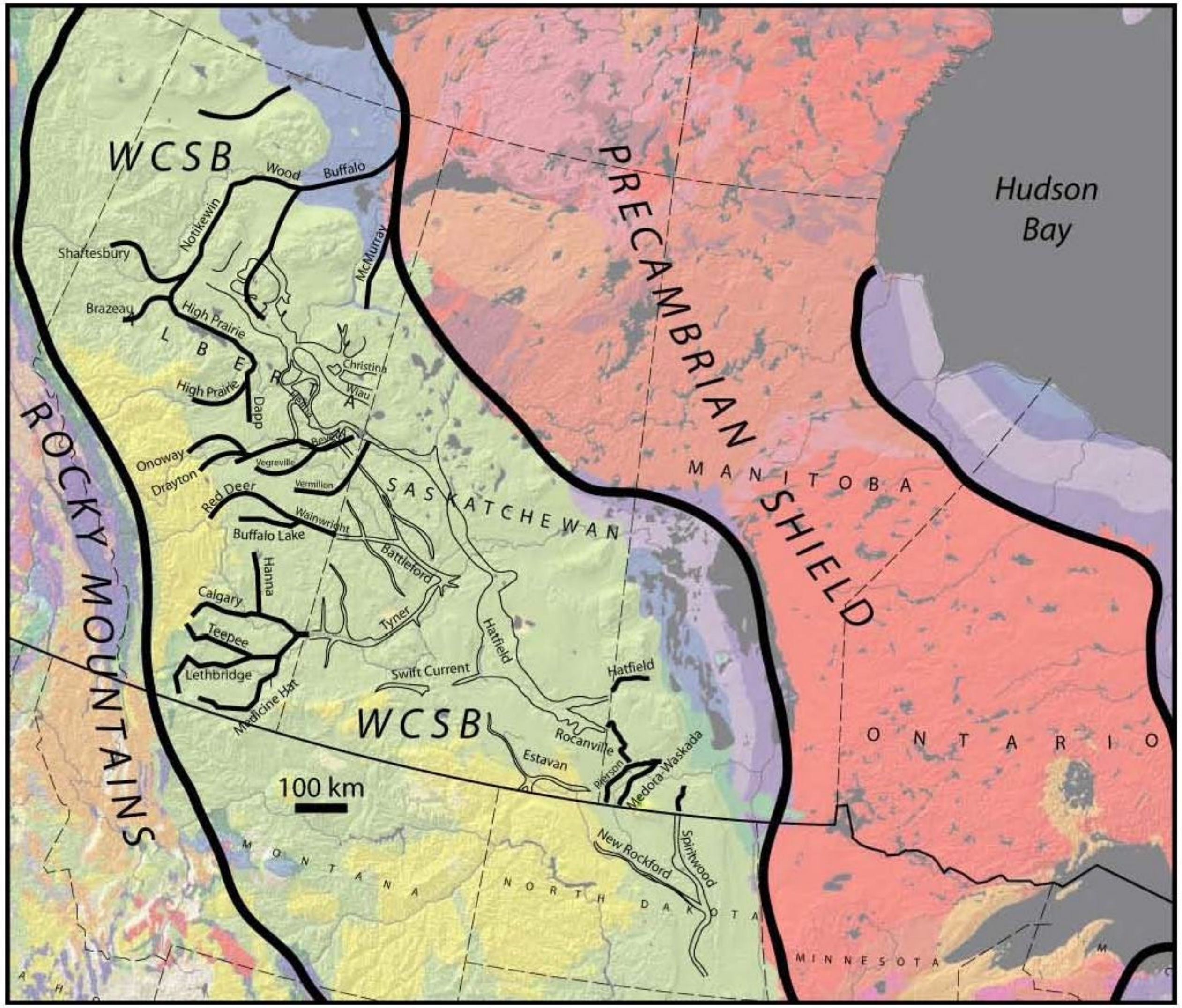
$K_2 > K_1$

$K_2 \rightarrow \infty$



Zone 1: K_H = 10 m/d
Zone 2
Zone 2
K_H = 1 m/d
K_H = 10 m/d
K_H = 100 m/d
Symbols: Drawdown
Dashed lines: Derivative
Drawdown (m)/Derivative (m)
Elapsed time (days)
0
0.5
1
1.5
2
2.5
3
3.5
0.01
0.1
1
Derivative
$K_2 < K_1$
$K_2 > K_1$

Buried valley aquifers: Northern Great Plains



Cummings, Russell and Sharpe (2011)

WCSB:
Western Canadian Sedimentary Basin

Idealized model for a buried channel aquifer

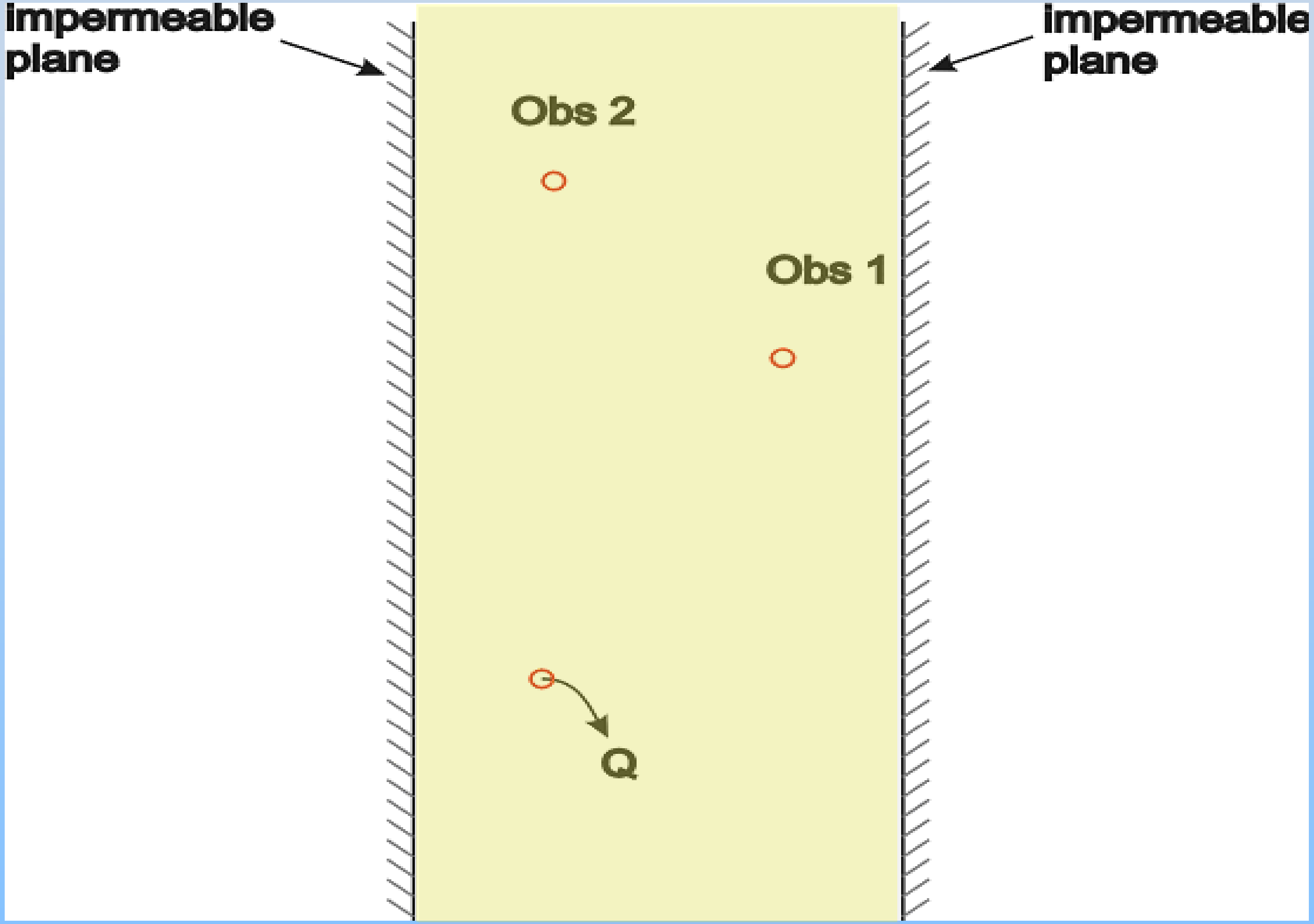
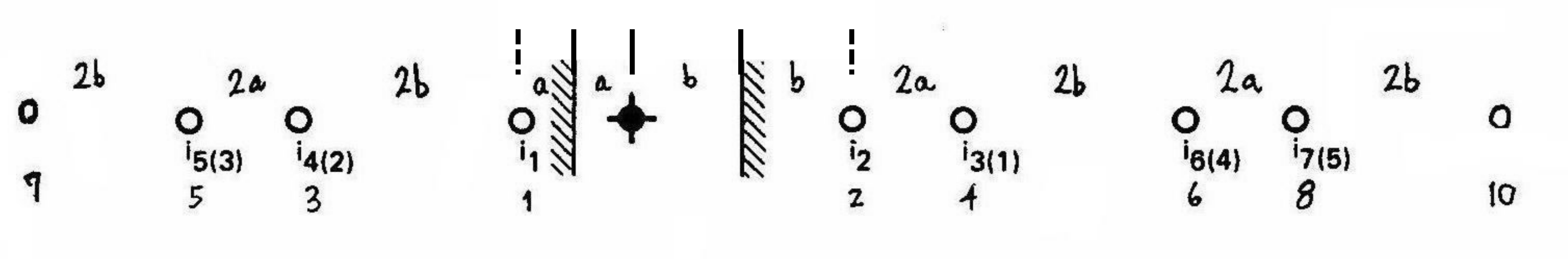


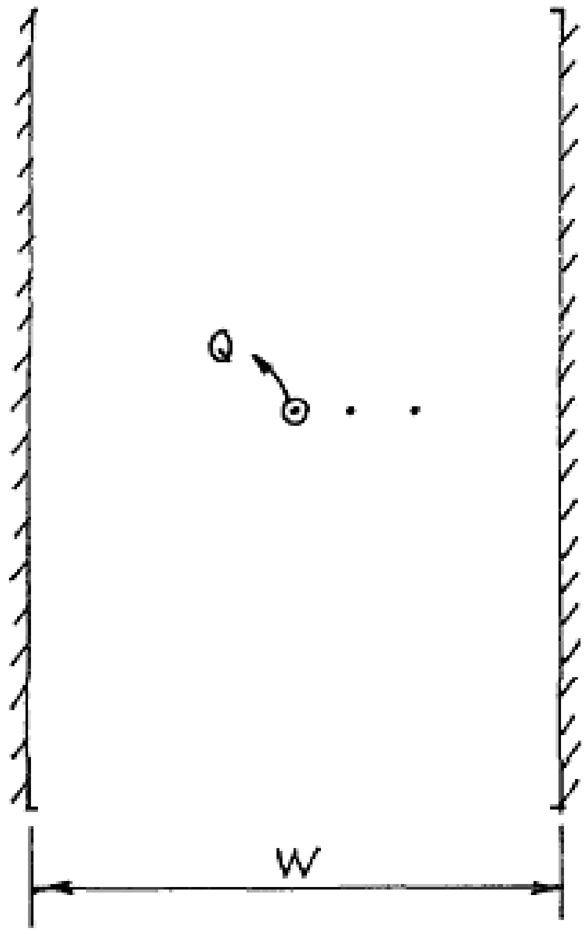
Image theory for two impermeable boundaries



$$s(r,t) = \frac{Q}{4\pi T}\left[W(u) + \sum_{i=1}^{N} W\left(A_{ri}^2 u\right)\right]$$

$$A_{ri} = \frac{r_i}{r}$$

Example analysis



T = 8.64 m^2/d

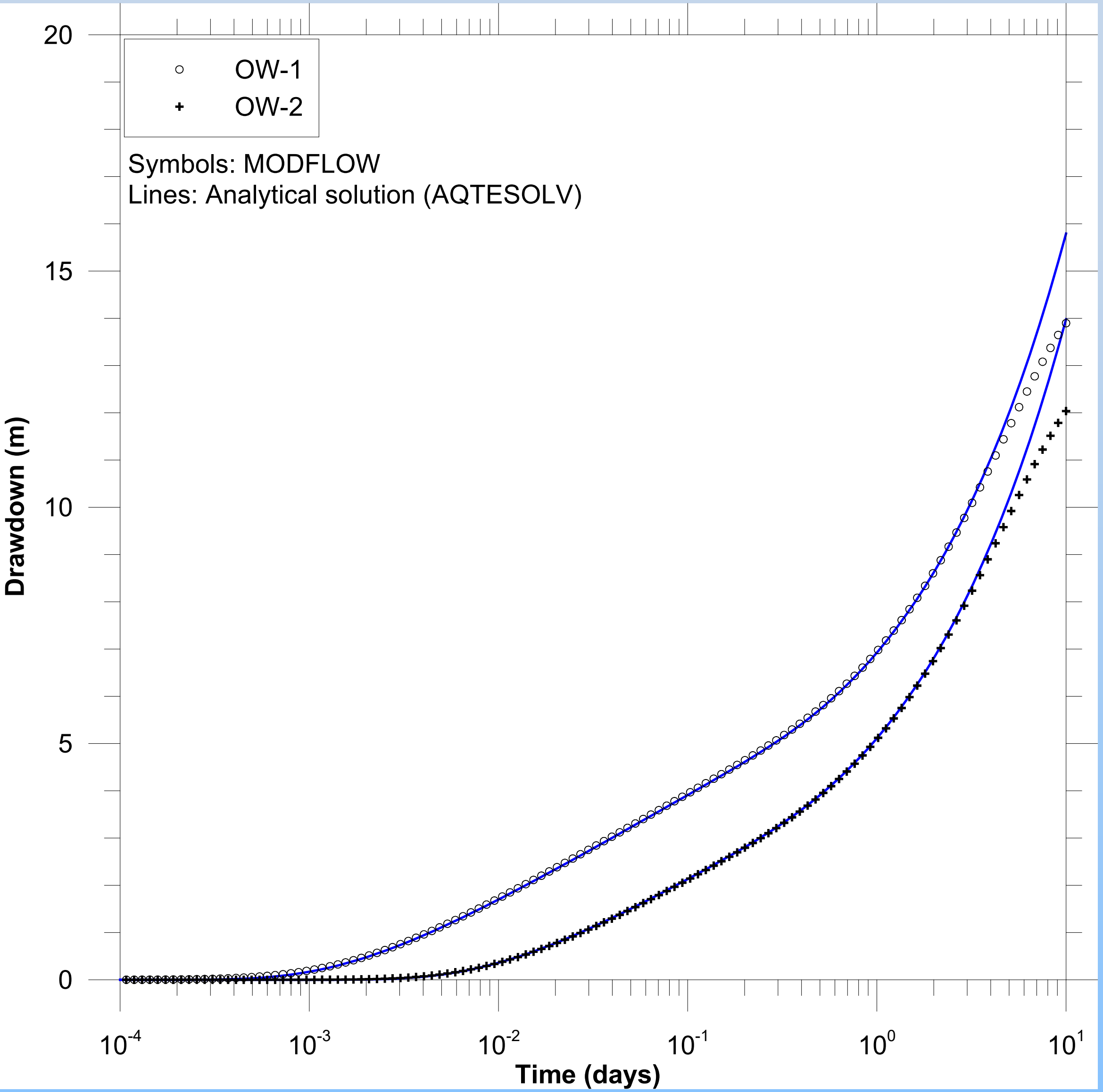
S = 1.0×10^{-4}

W = 510 m

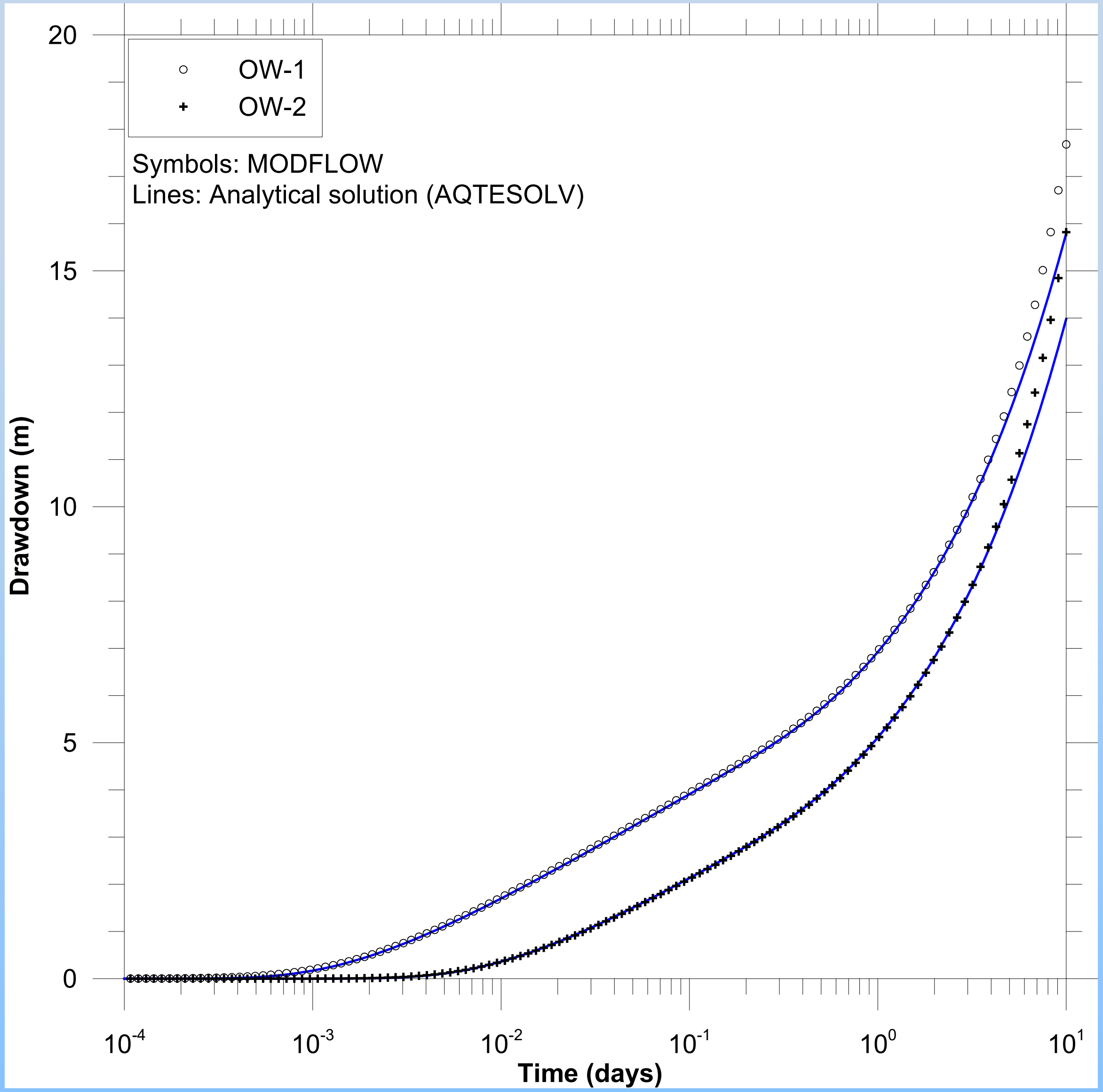
x_{OW1} = 20 m

x_{OW2} = 50 m

Simulation #1
Constant-head conditions along N-S boundaries



Simulation #2
No-flow conditions along N-S boundaries



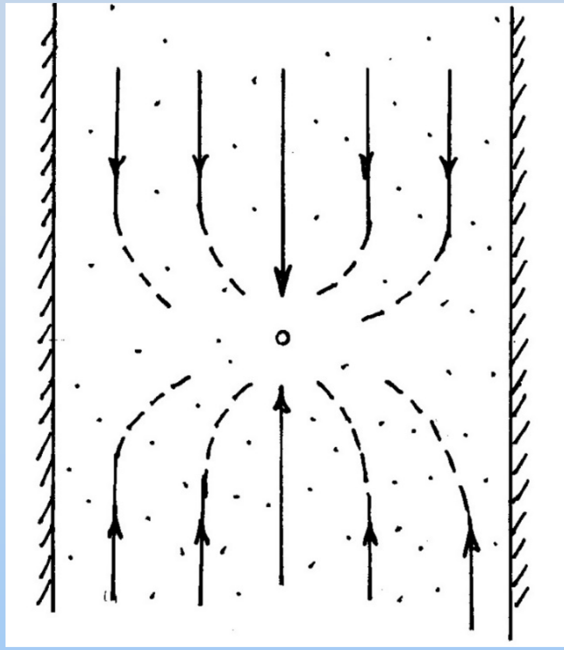
Keys to the analyses of pumping tests in buried channel aquifers

- Recognition that the aquifer is a buried channel; and

- Identification of the portion of the response that should be considered for the estimation of the transmissivity (i.e., the portion of the response that is not affected by the boundaries.

Late time response

~Straight-line flow



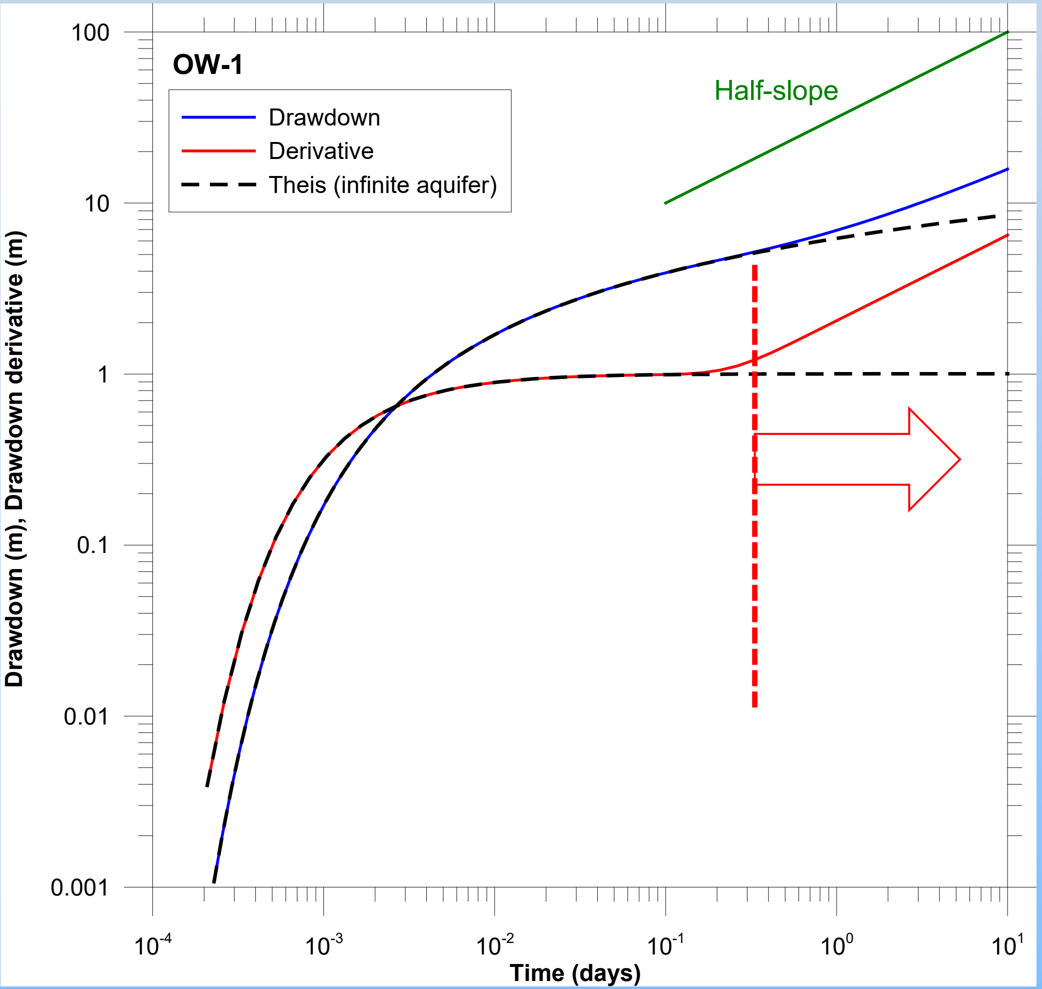
$$s(r,t) \cong \frac{Q}{W}\frac{1}{2T}\left[\left(\frac{4Tt}{\pi S}\right)^{1/2} - y\right]$$

$$s(r,t) \sim t^{1/2} \rightarrow \log[s] \sim \frac{1}{2}\log[t]$$

$$D_t \sim t^{1/2} \rightarrow \log[D_t] \sim \frac{1}{2}\log[t]$$

Identification of a buried valley

Log-log plot with derivative: Late time half-slope

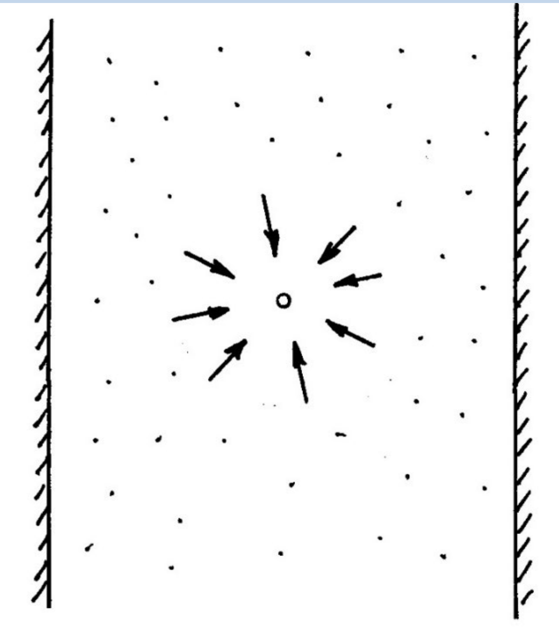


Drawdown

Derivative

Early time response

Radial flow



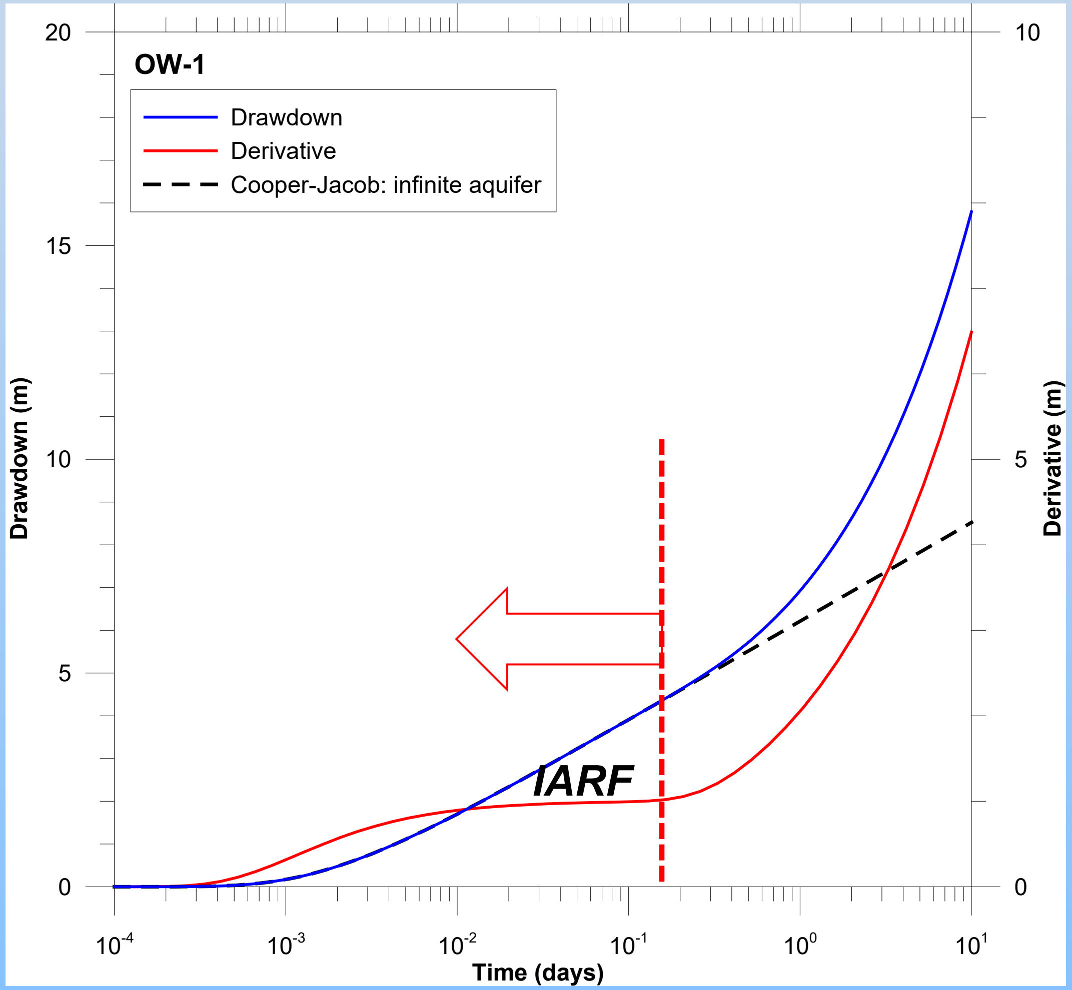
$$s(r,t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) \cong \frac{Q}{4\pi T}\left[-0.5772 + \ln\left\{\frac{4Tt}{r^2 S}\right\}\right]$$

$\rightarrow s(r,t) \sim \log\{t\}$

$D_t \rightarrow \text{constant}$

Estimation of transmissivity (pre-boundary effects)

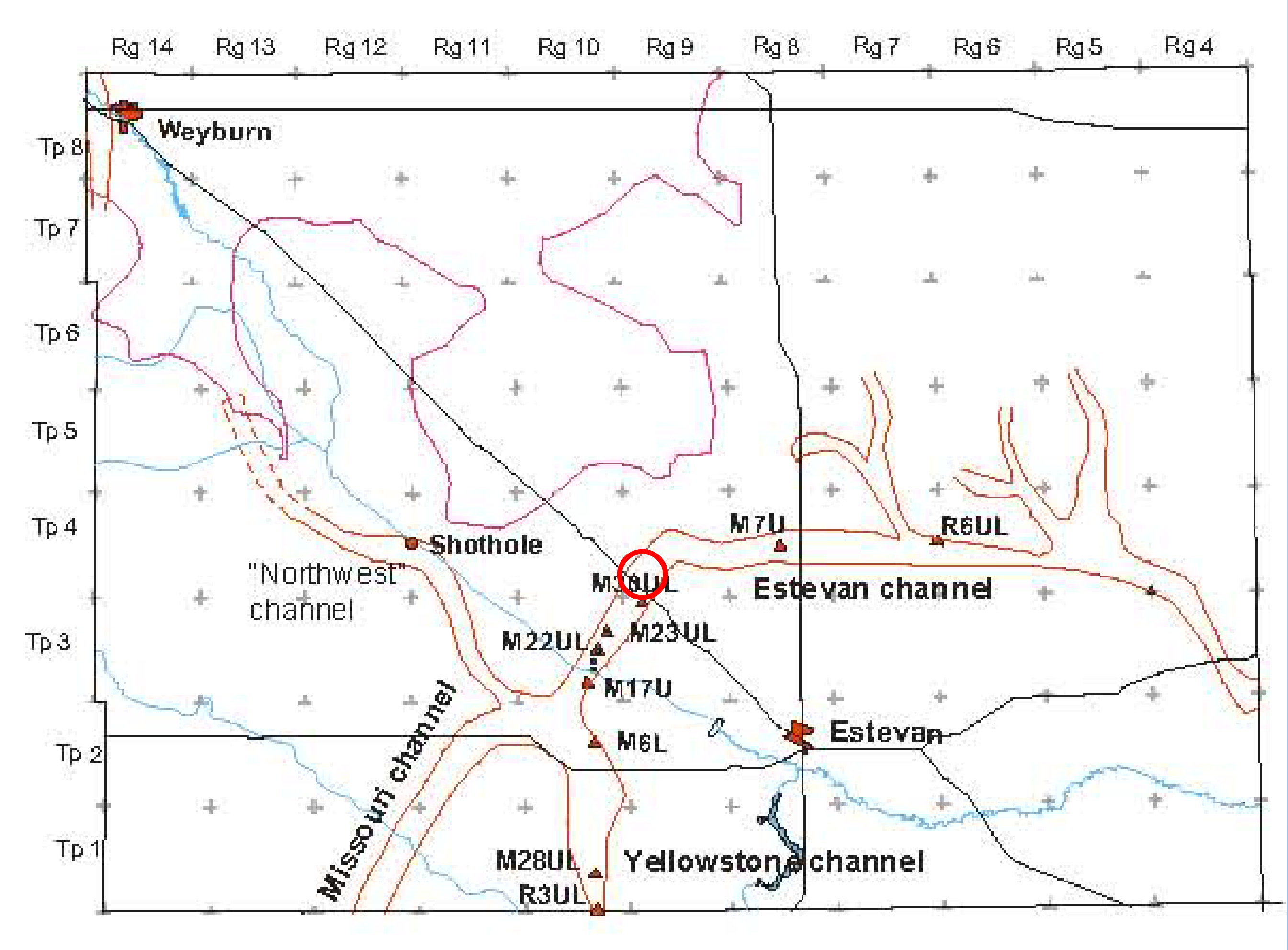
Semilog plot with derivative: Early time plateau



Case study:
Estevan, Saskatchewan (1964)

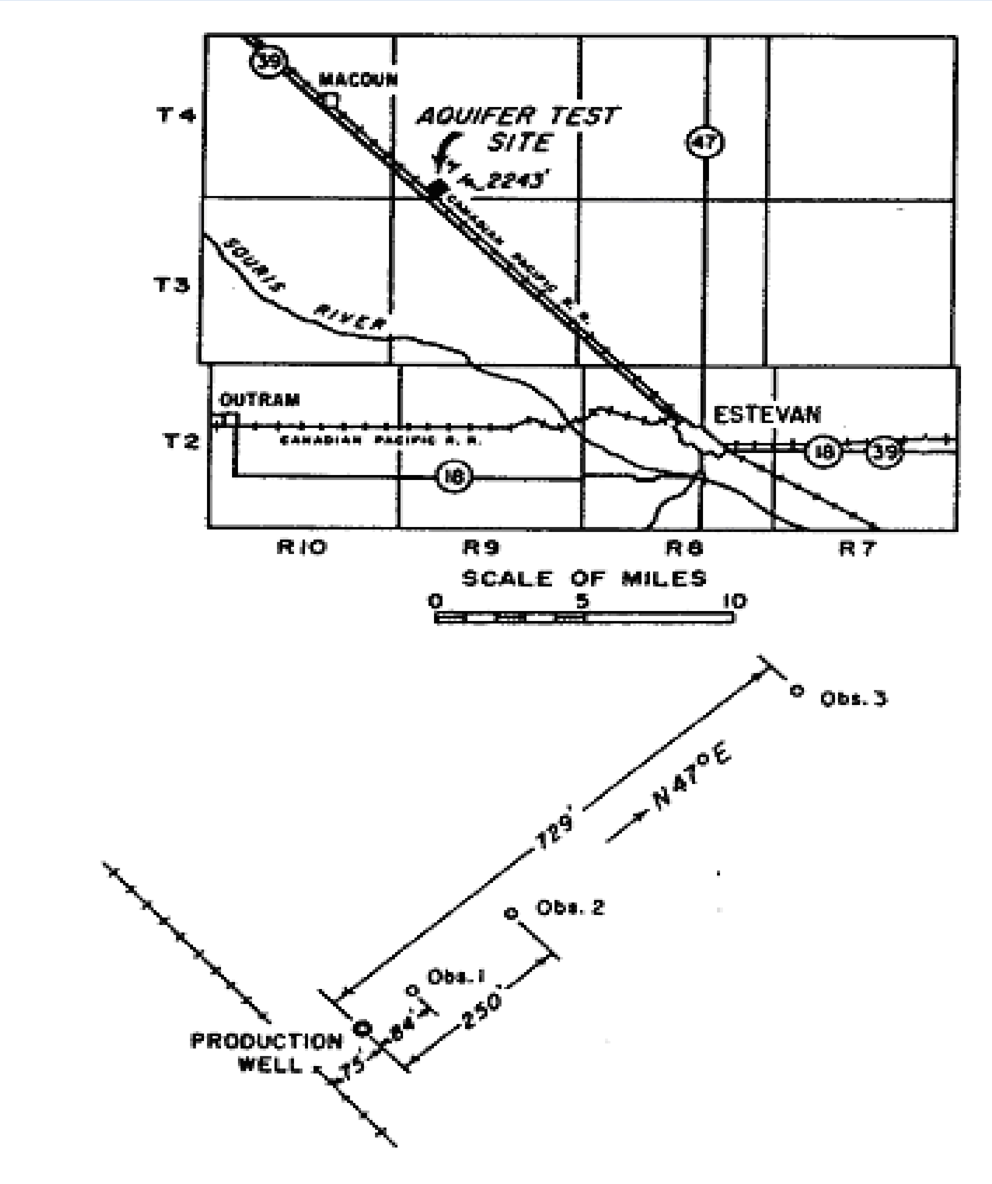
SWA Estevan Observation well, Saskatchewan

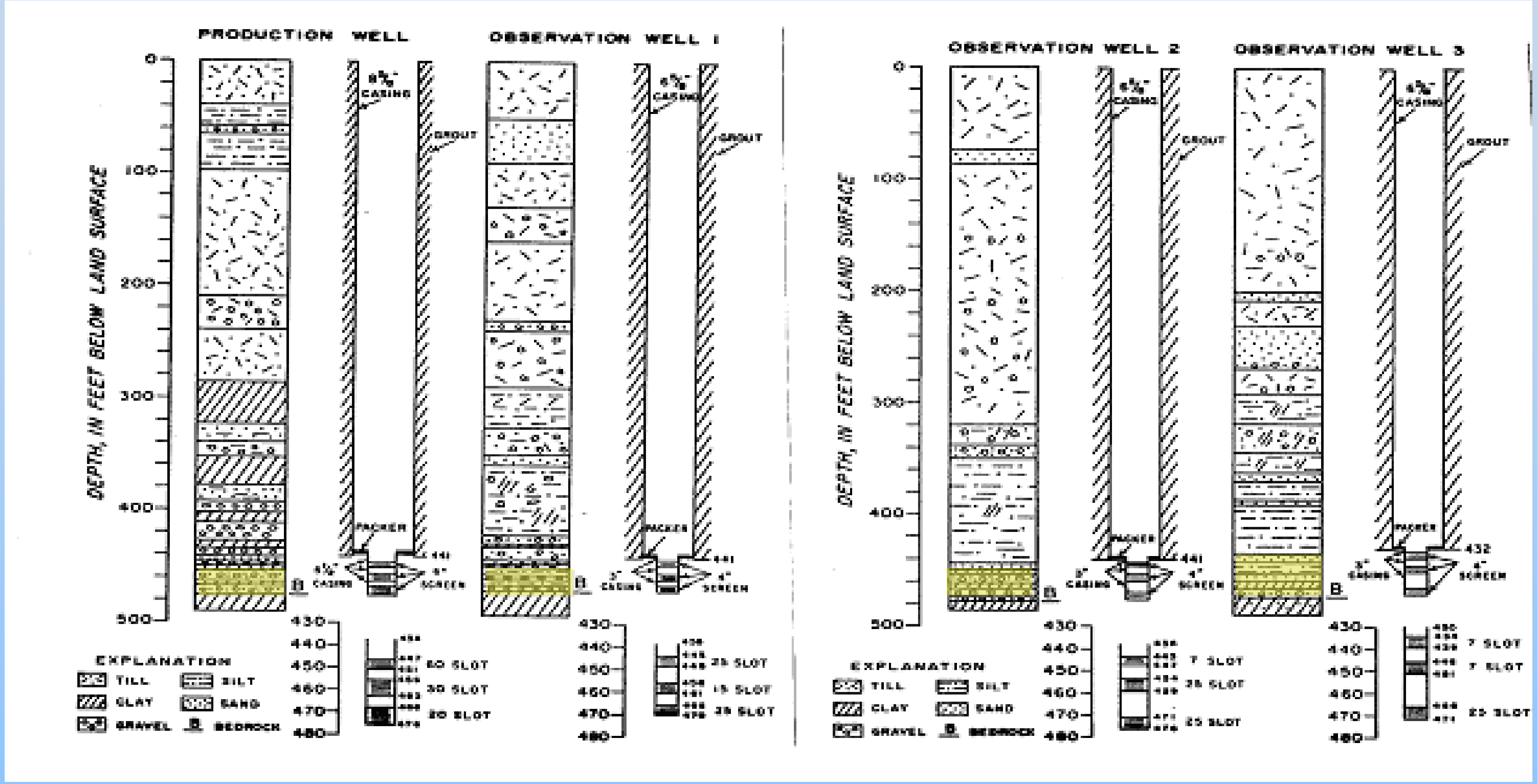
Estevan Valley aquifer system



Maathuis and van der Kamp (2003)

1964 pumping test (Walton,1970)




PRODUCTION WELL
OBSERVATION WELL 1
OBSERVATION WELL 2
OBSERVATION WELL 3
DEPTH, IN FEET BELOW LAND SURFACE
CASING
GROUT
PACKER
SCREEN
EXPLANATION
TILL
SILT
CLAY
SAND
GRAVEL
BEDROCK
30 SLOT
20 SLOT
25 SLOT
15 SLOT
7 SLOT

Original analysis: Walton (1970)

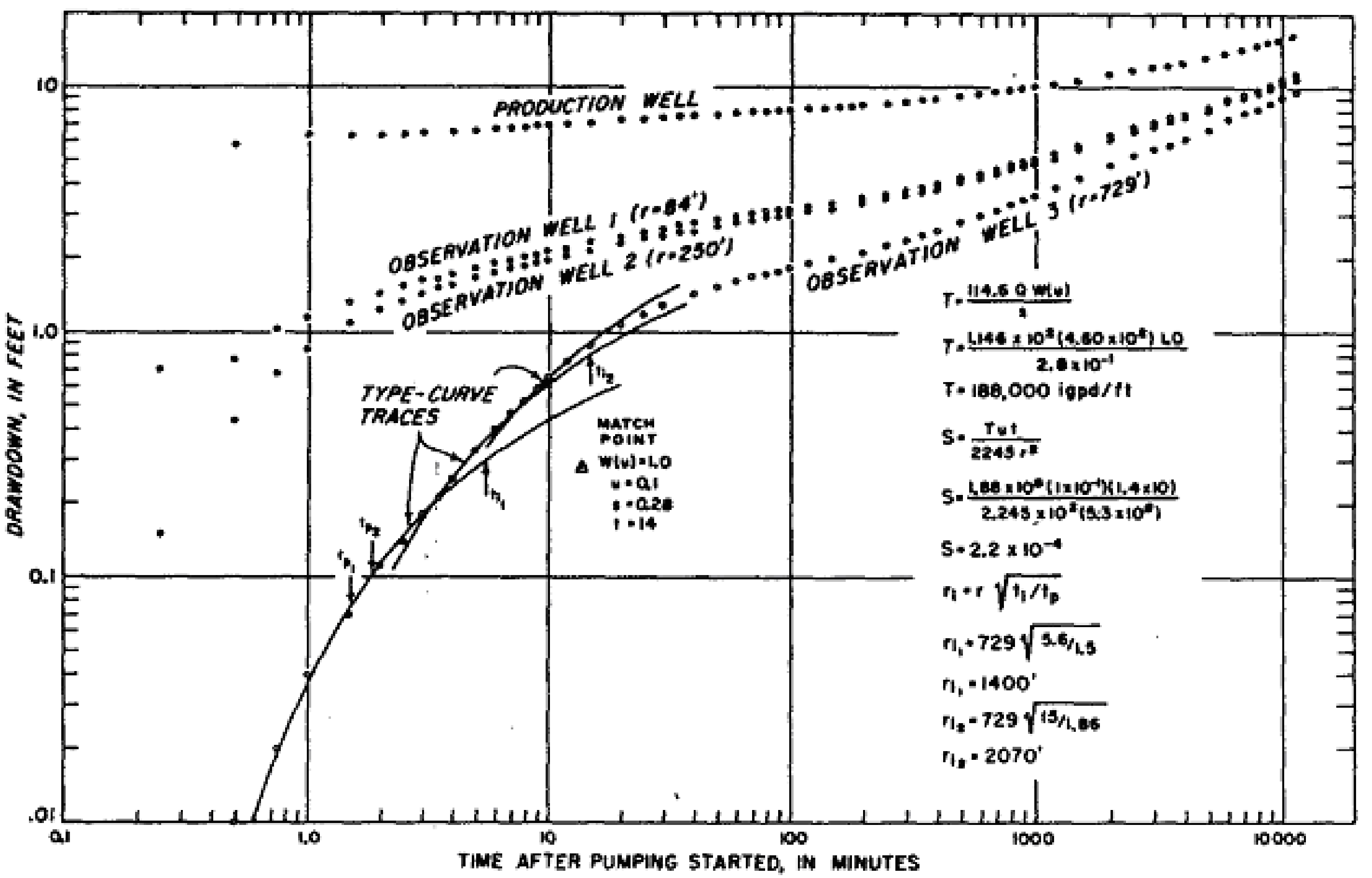


Fig. 4.42 Time-drawdown graphs for wells used in aquifer test near Estevan, Saskatchewan, Canada. (*From Walton, 1965c.*)

Results of Walton (1970) analysis:

T = 30,220 ft²/d

$S = 2.2\times10^{-4}$

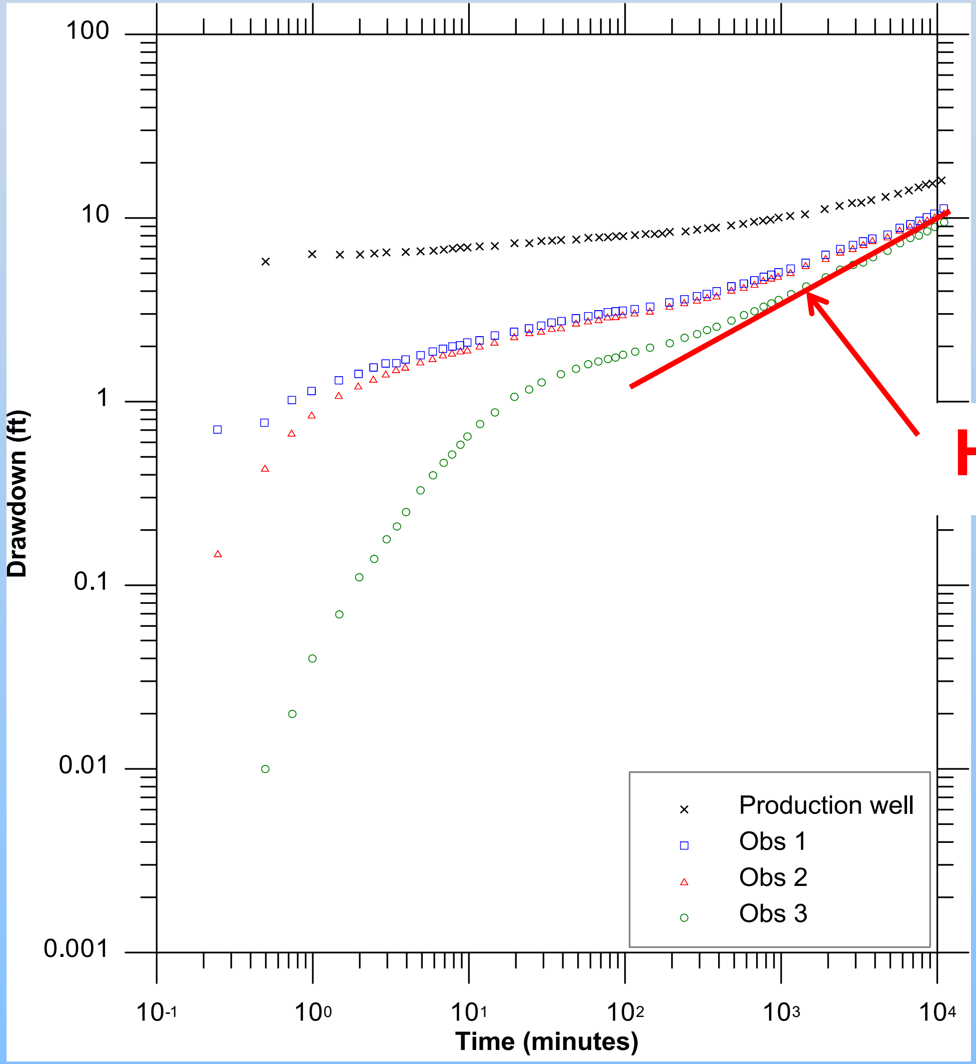
Objections:

- Recognition of the presence of boundaries, but no attempt to account for them in the analysis.
- Most of the data are neglected.
- No idea whether the estimated T is representative.

Alternative analysis

1. Diagnosis of a buried channel aquifer
2. Estimation of transmissivity from an *appropriate* Cooper-Jacob analysis, based on the detection of the infinite-acting radial flow regime
3. Analysis as a buried channel aquifer

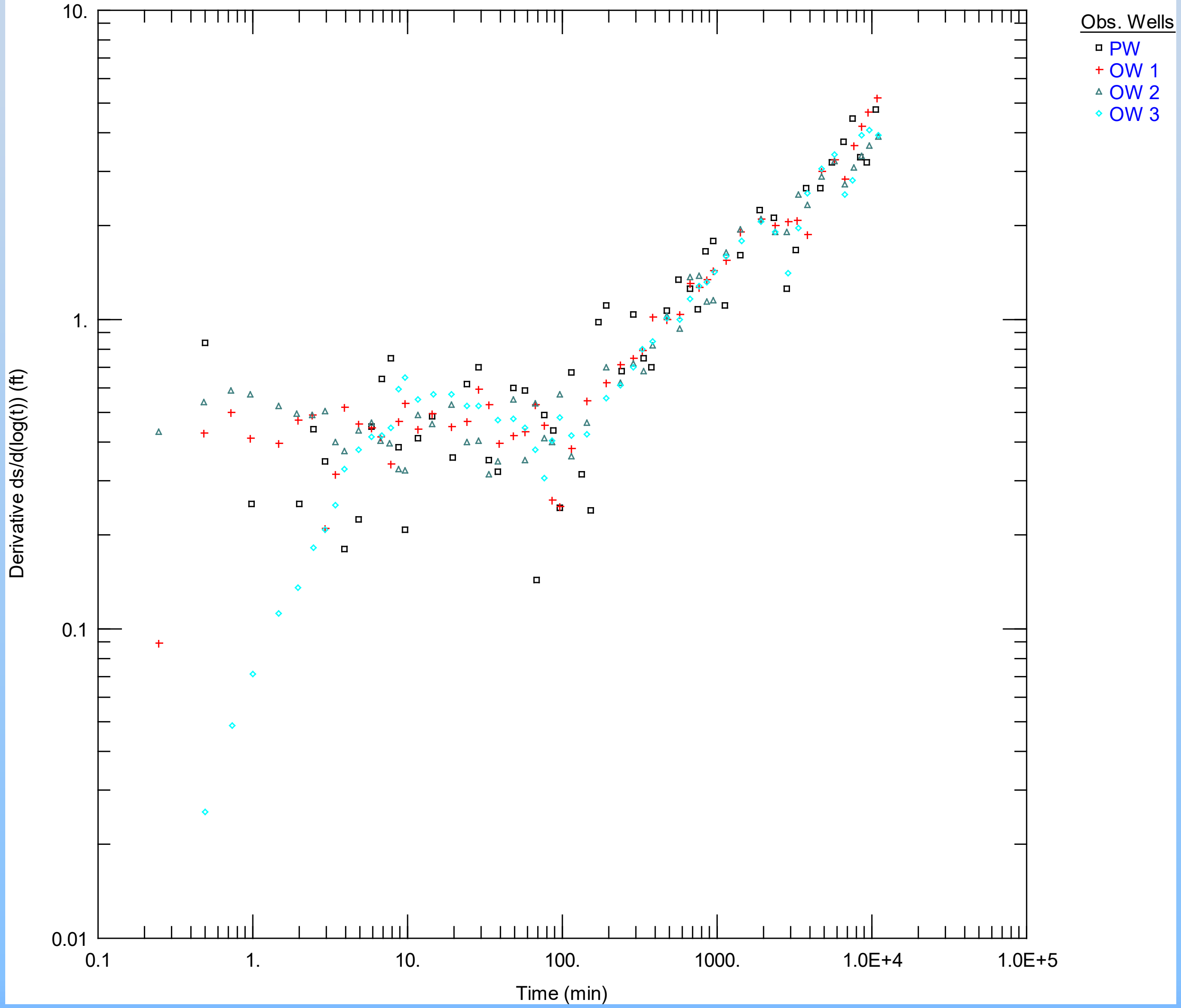
1) Diagnosis of buried valley: Log-log plot



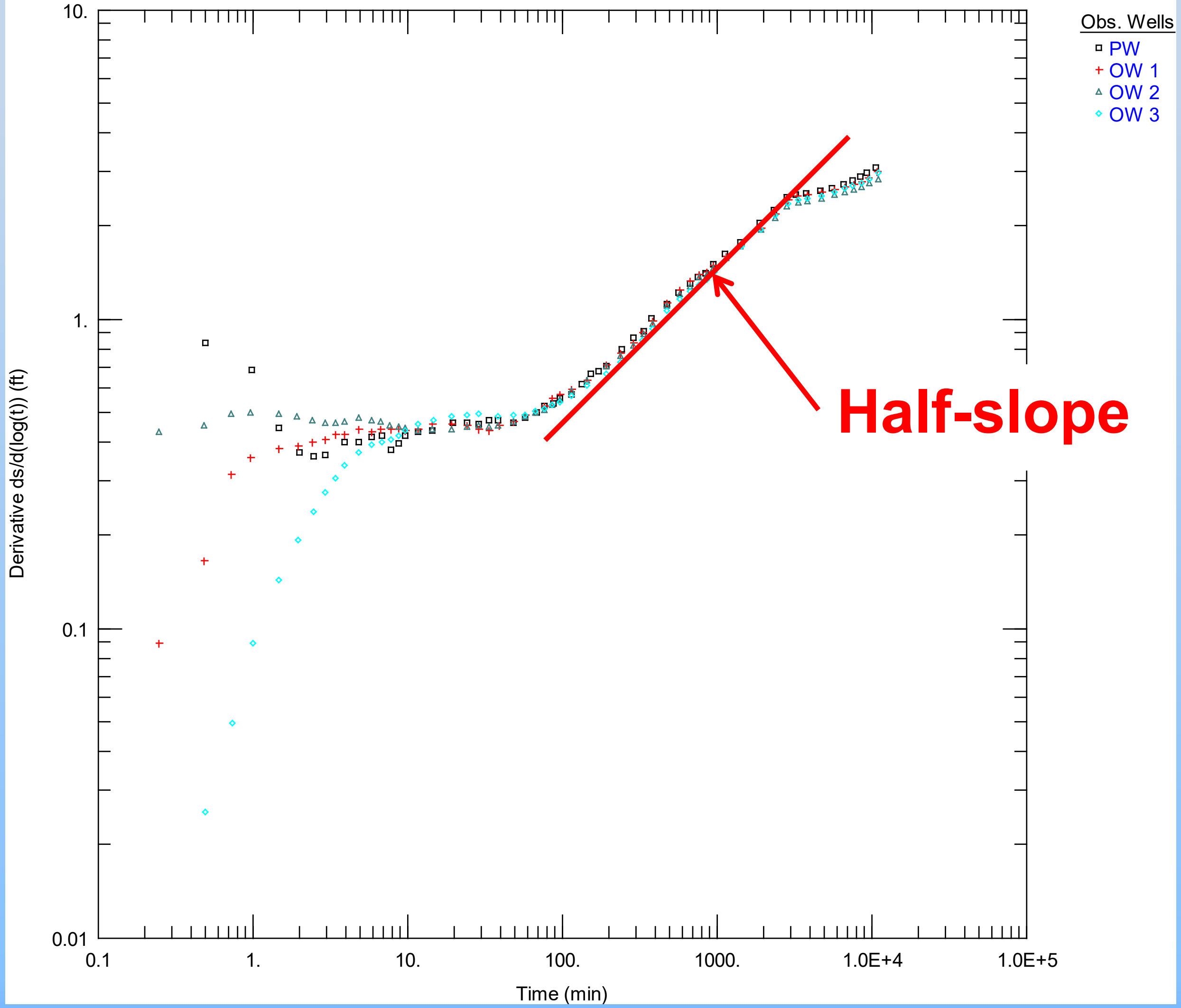
Late-time straight lines

Half-slope

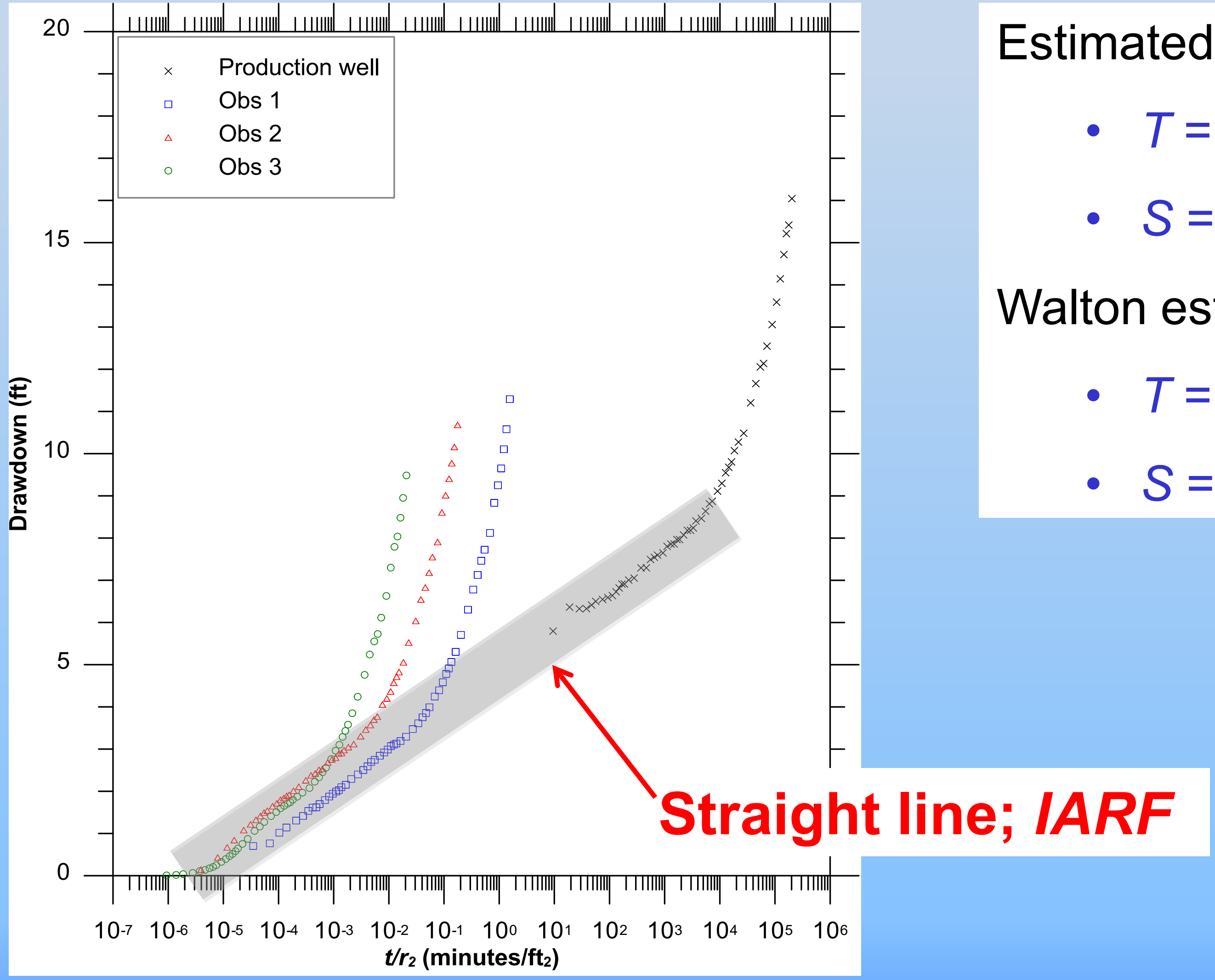
Further diagnosis: Derivative plot (raw)



Derivative plot (smoothed)



2) Estimation of transmissivity: Semilog composite plot



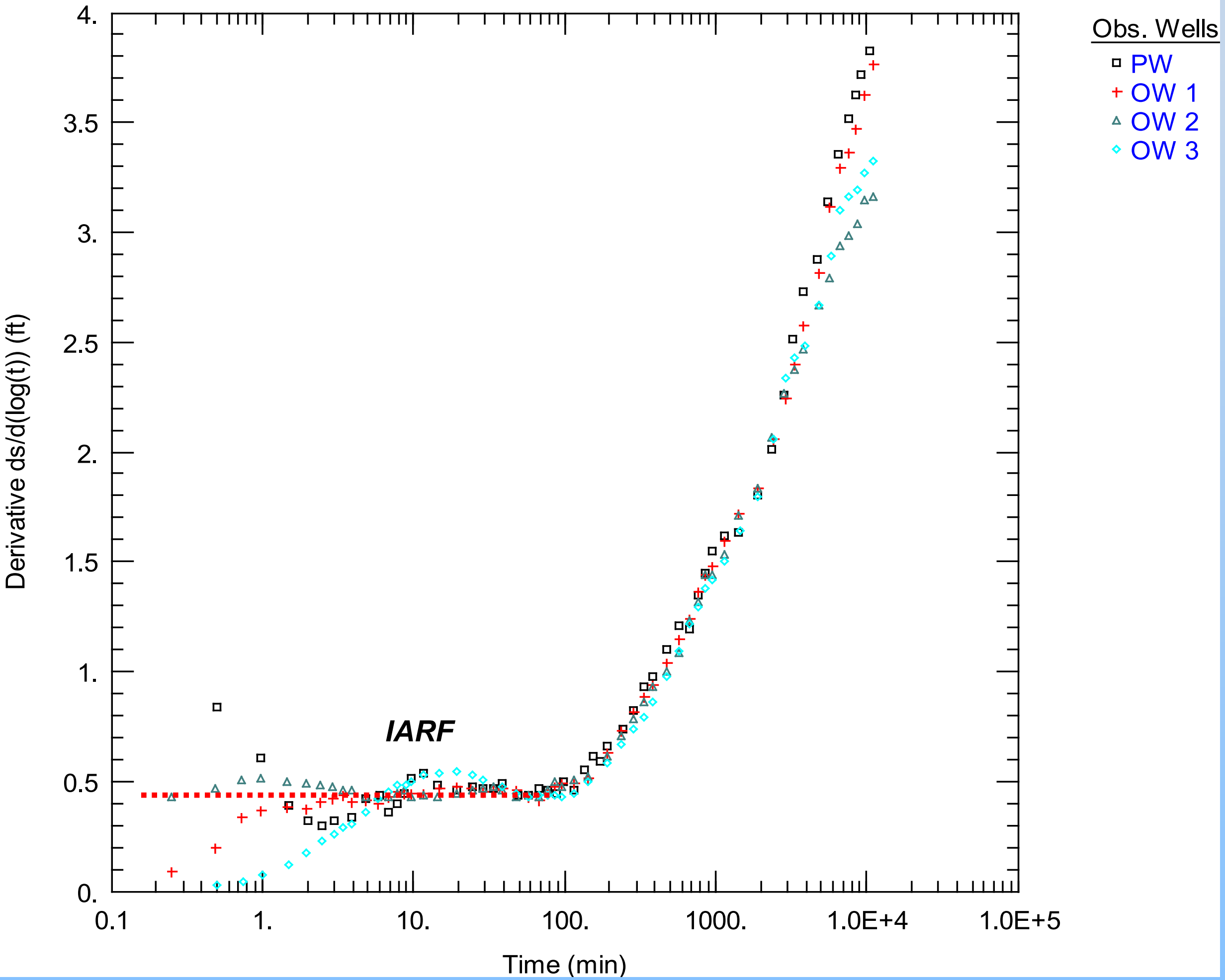
Estimated parameters:

- T = 21,000 ft²/d
- $S = 1.0\times10^{-4}$

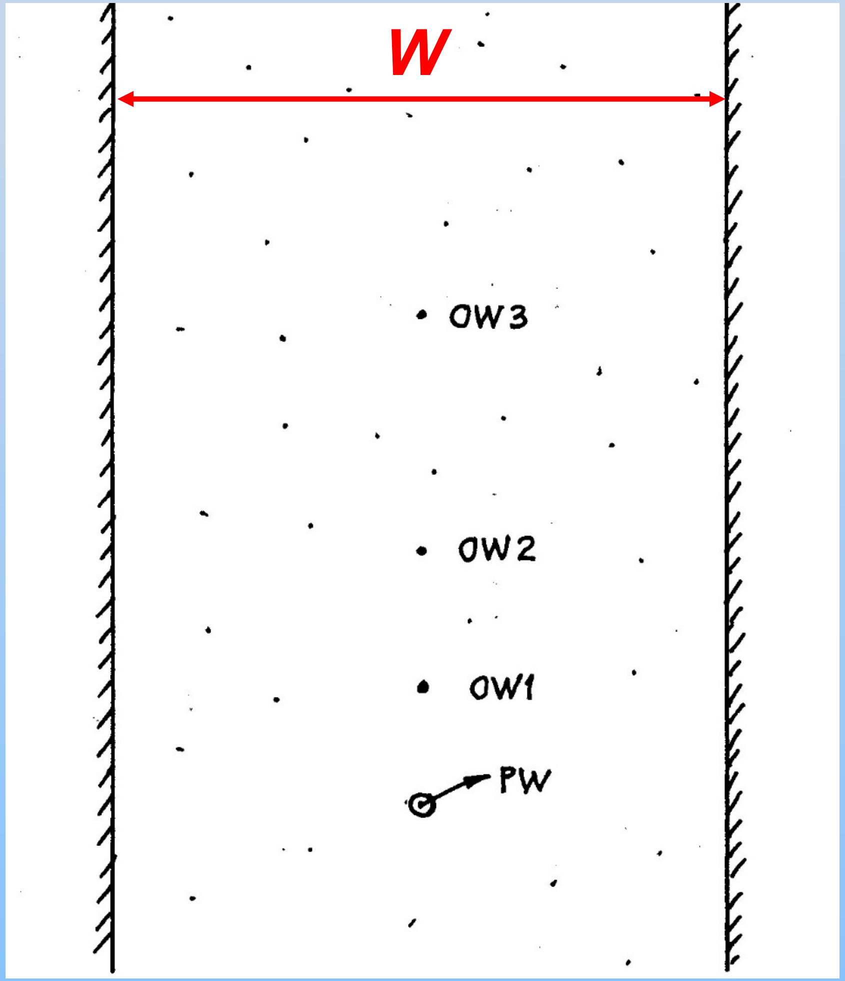
Walton estimates:

- T = 30,220 ft²/d
- $S = 2.2\times10^{-4}$

Confirmation of the identification of the IARF period



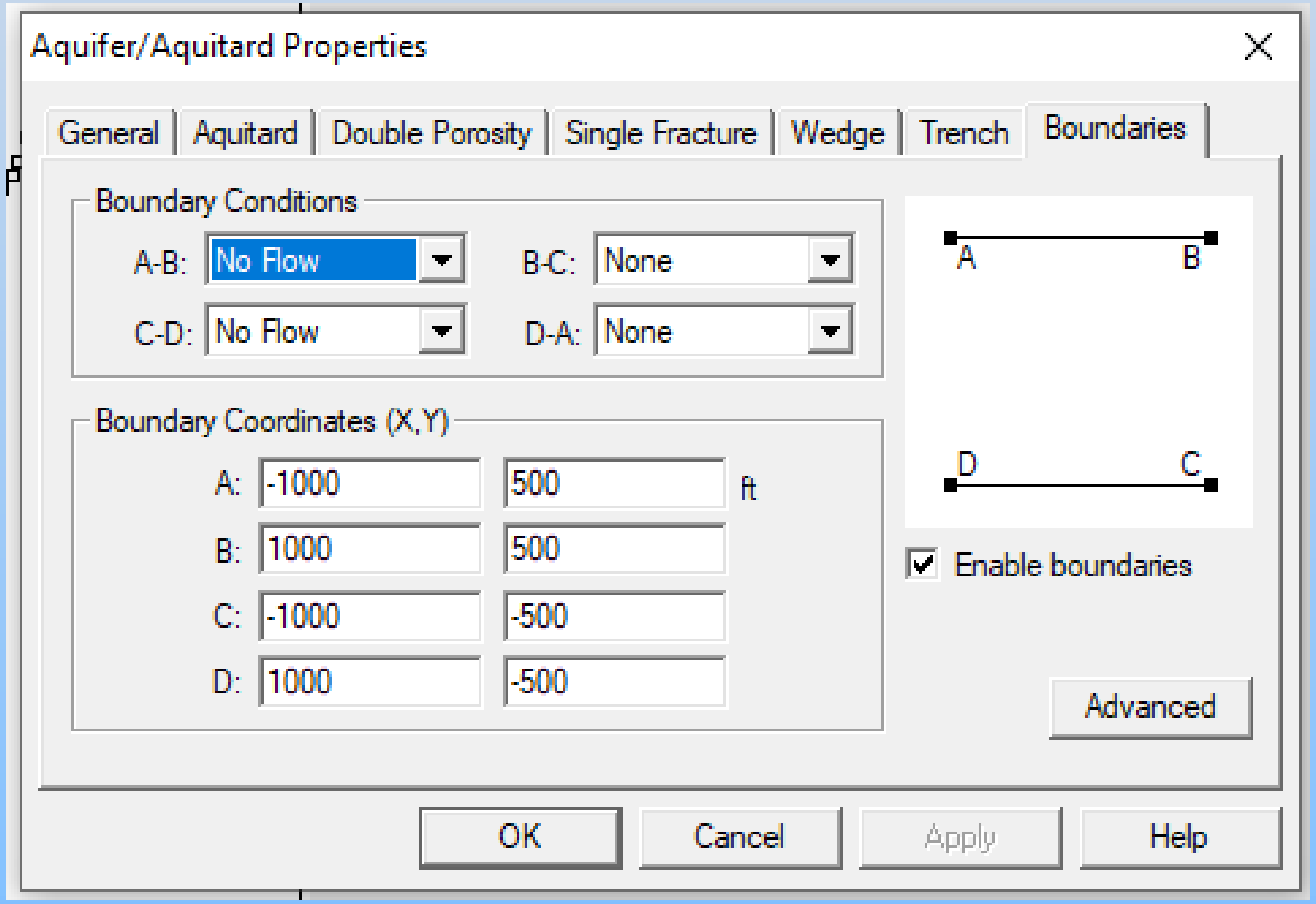
3) Complete analysis: Channel aquifer

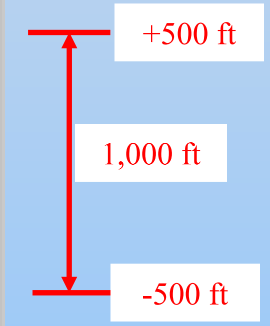


Implementation of channel model in AQTESOLV

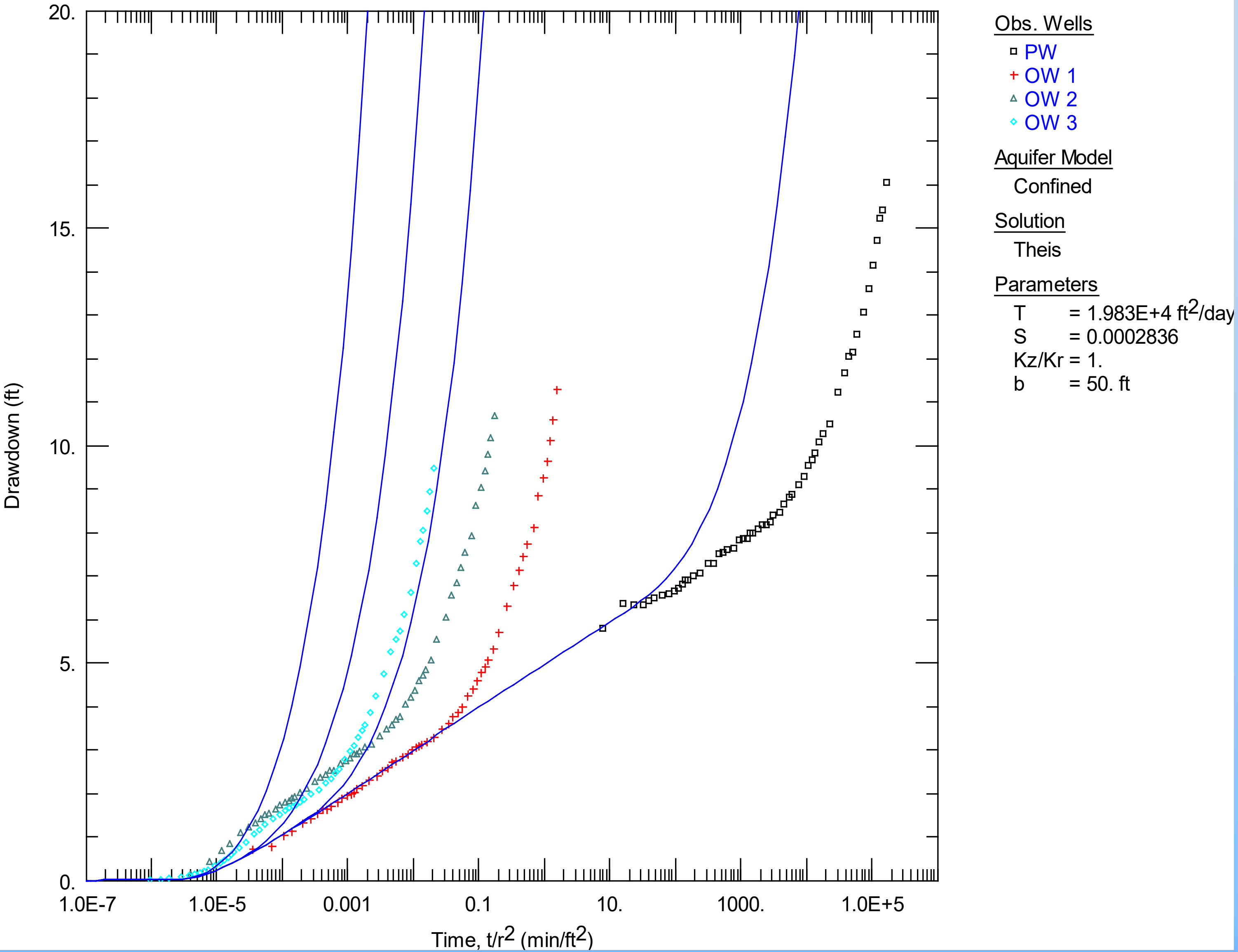
Trial #1

W = 1000 ft



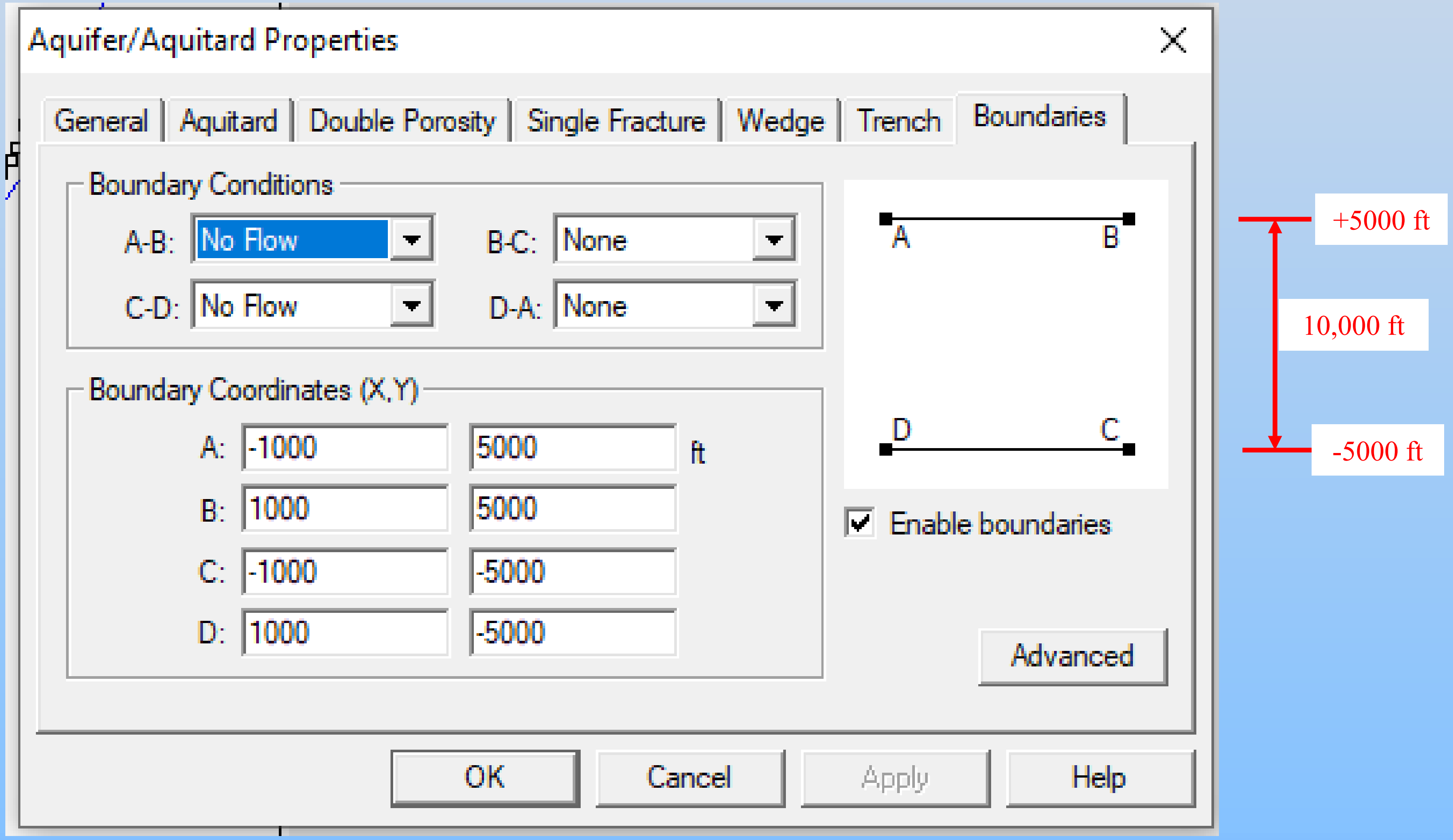


Trial #1: *W* = 1,000 ft

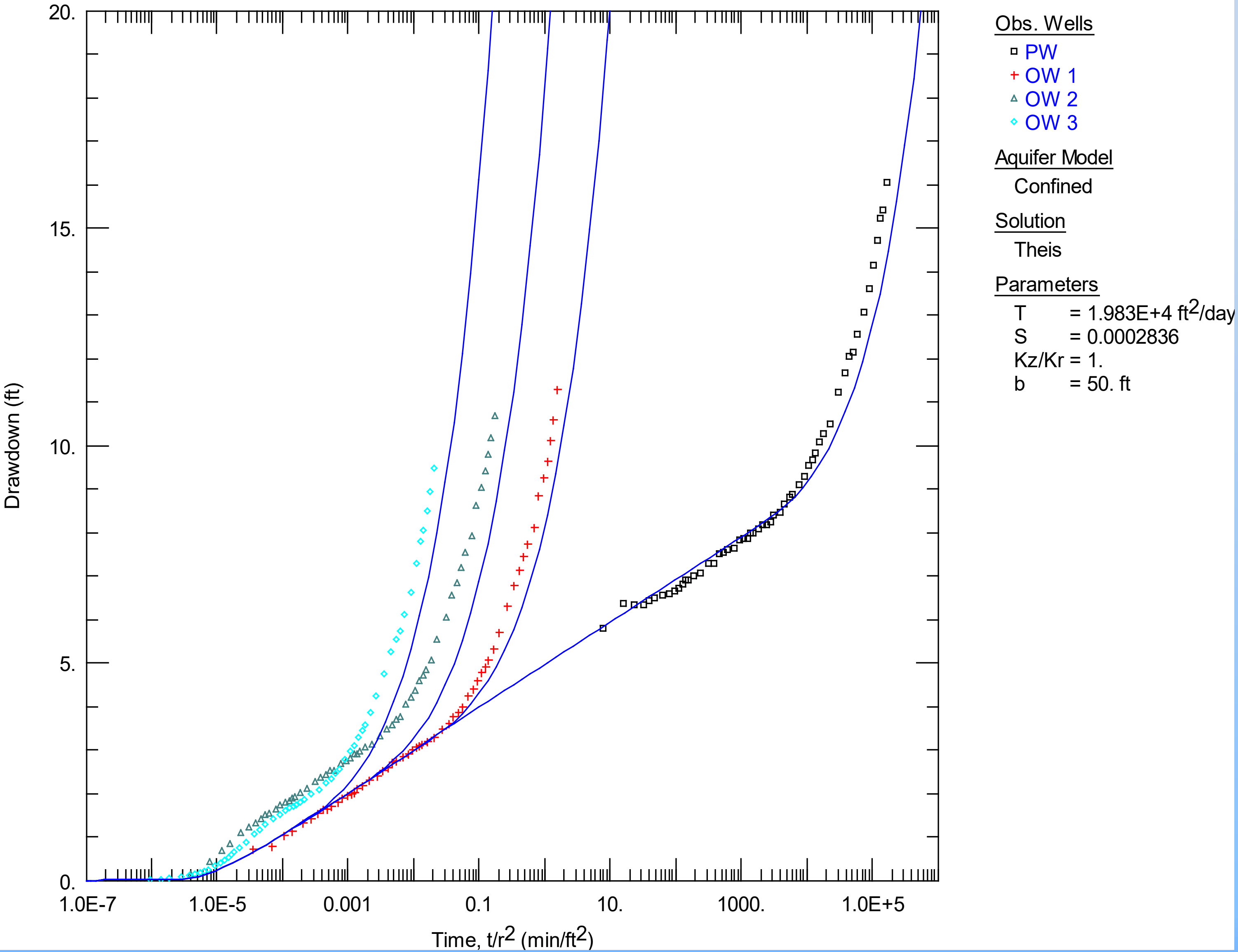


Trial #1

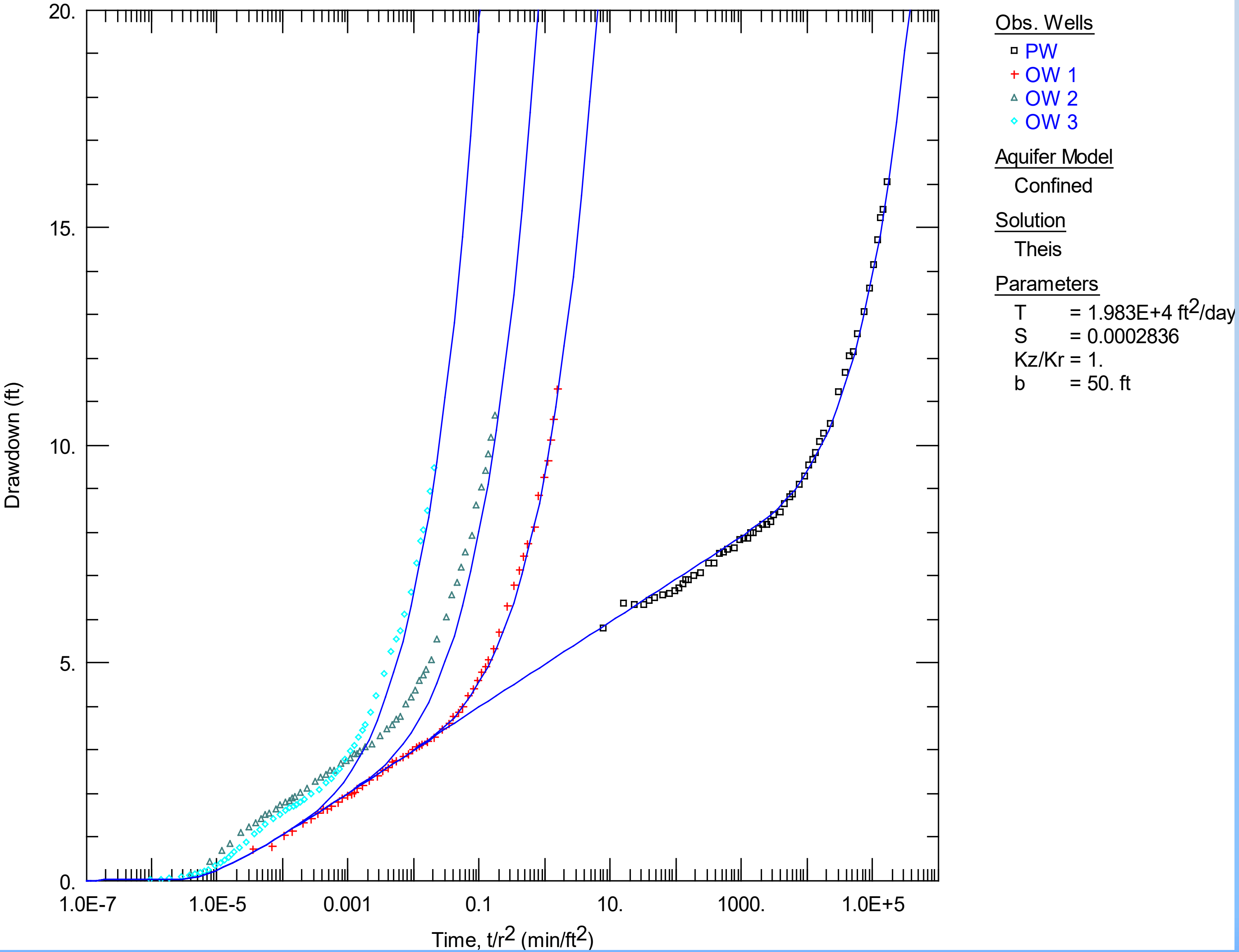
W = 10,000 ft



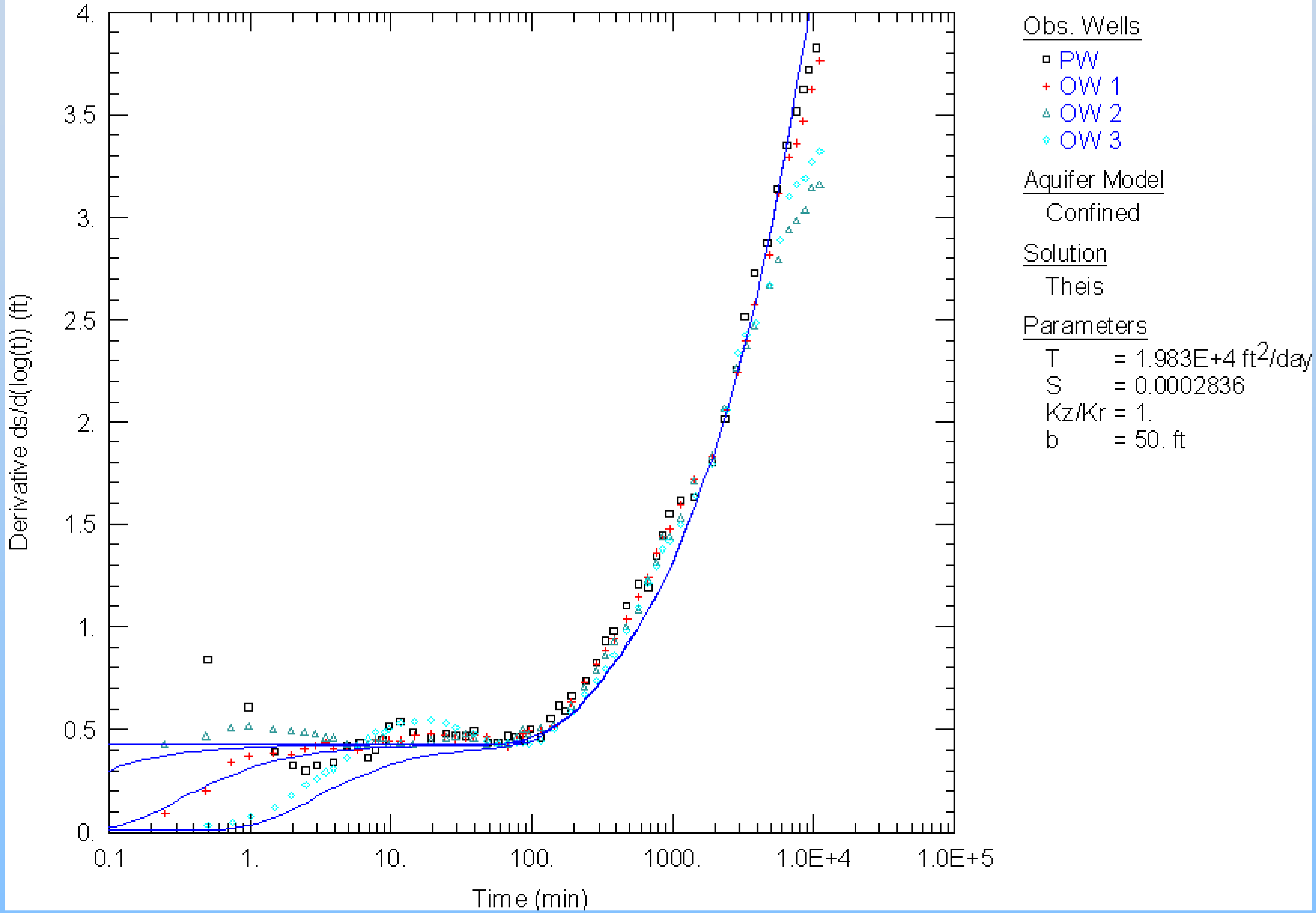
Trial #2: *W* = 10,000 ft



Trial #3: *W* = 8,000 ft



Check on conceptual model with derivative



Take-home points

1. For linear systems, boundaries can be accommodated with superposition.
2. Use both log-log and semi-log plots to enhance the diagnostic value of the analysis.
3. Plot the drawdown derivative.
4. Take advantage of computer-assisted methods to interpret pumping tests in channel aquifers.